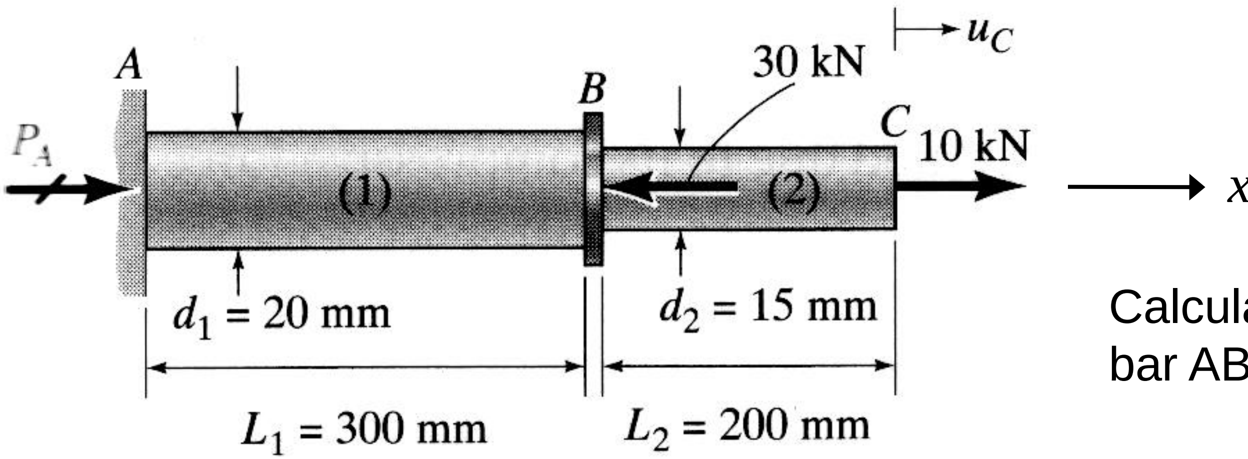
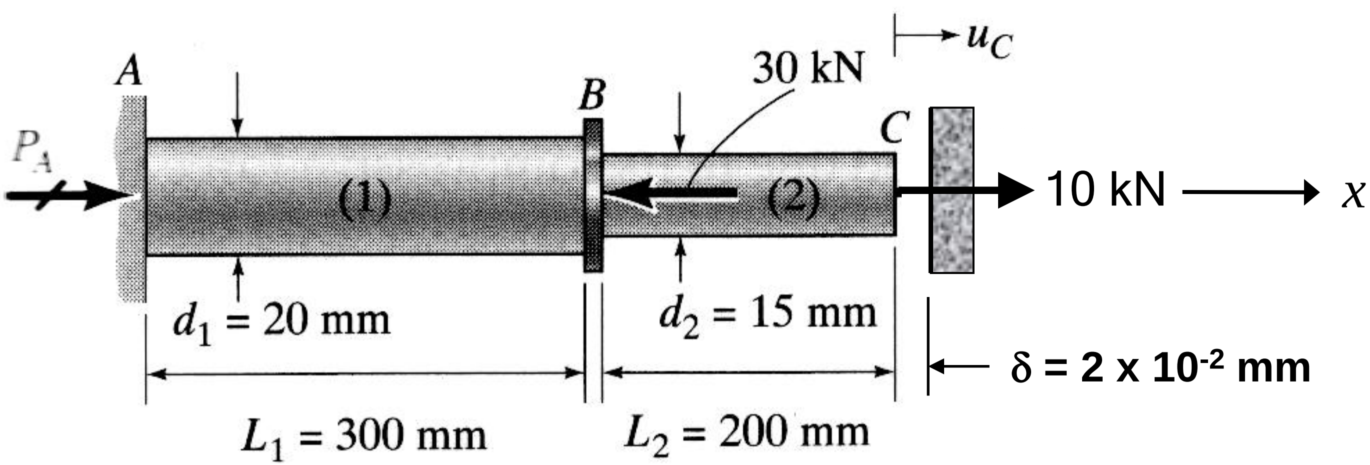
One-Dimensional Problems

We wish to use FEM for solving the following problems:



Calculate displacement of bar ABC, take E = 200GPa

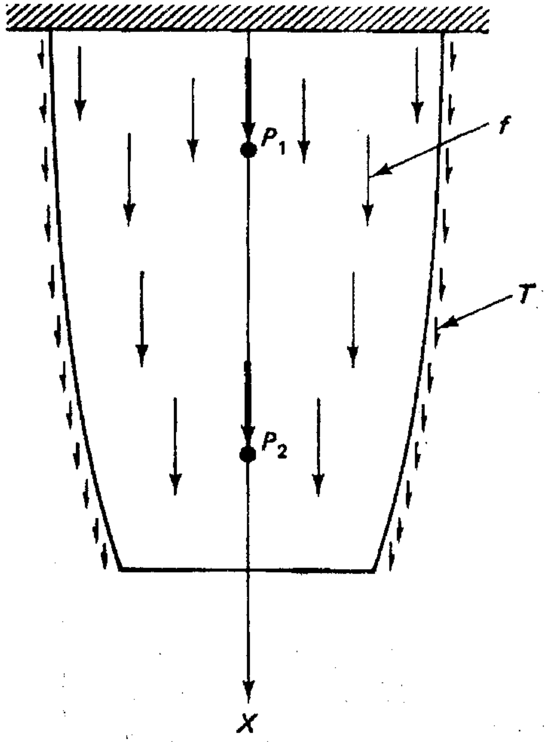


3-1 Objectives

1. To develop a system of linear equations for one-dimensional problem.
2. To apply FE method for solving general problems involving bar structures with different support conditions.

3-2 General Loading Condition

Consider a non-uniform bar subjected to a general loading condition, as shown.



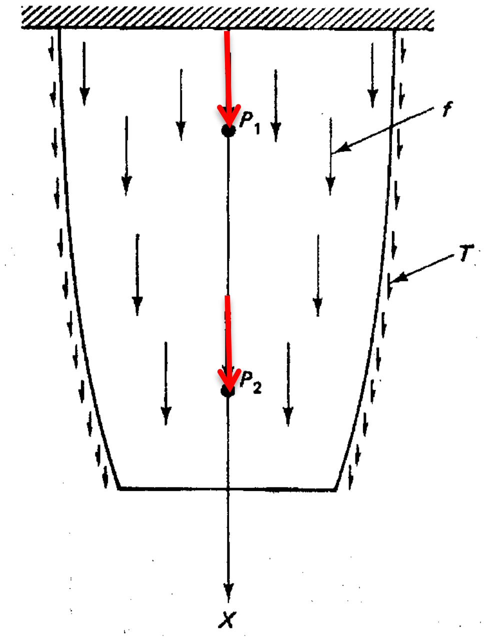
Note: The bar is constrained by a fix support at the top and is free at the other end. The positive x-direction is taken downward.

Types of Loading

a) Body force, f

Distributed force per unit volume (N/m^3)

Example: self-weight due to gravity



b) Traction force, T

Force per unit area (N/m^2)

For a 1-D problem,

$$T = \left(\frac{\text{force}}{\text{area}}\right) \times (\text{perimeter of area})$$

Examples: Frictional forces, Viscous drag, and Surface shear.

c) Point load, P_i

Concentrated load (in Newton) acting at any point *i*.

3-3 Finite element Modeling

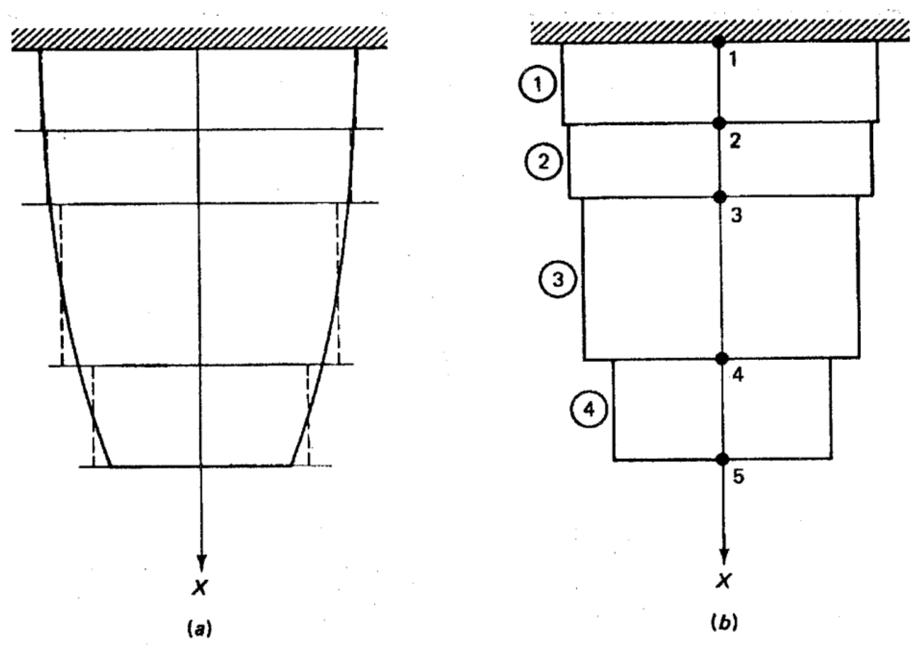
3-3-1 Element Discretization

The first step is to subdivide the bar into several sections – a process called discretization.

Note: The bar is discretized into 4 sections, each has a uniform cross-sectional area.

The non-uniform bar is transformed into a stepped bar.

We will use the stepped bar as a basis for developing a finite element model of the original non-uniform bar.



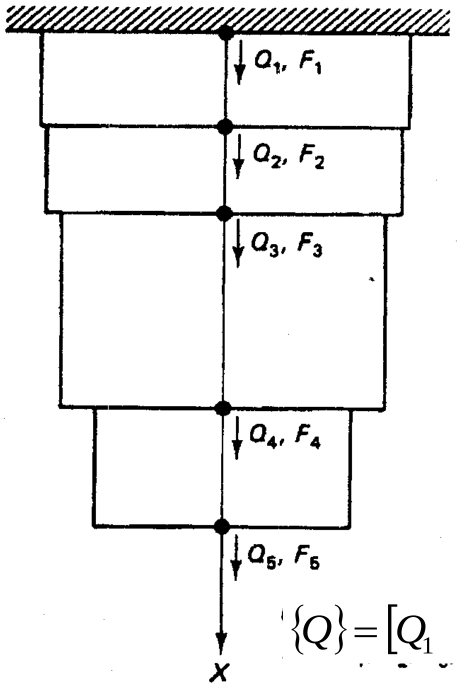
3-3-2 Numbering Scheme

To analyze the stepped bar systematically, a global numbering scheme is assigned as shown. The *x*-direction is considered as the global coordinate direction.

Note:

F_1, …, F_5 represent global forces acting on the points connecting all sections of the stepped bar.

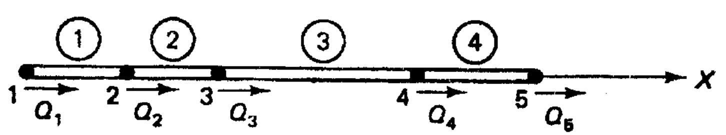
Q_1, …, Q_5 represent global displacements of the points resulting from the forces acting on these points.



$$\{Q\} = [Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_5]^T$$

$$\{F\} = [F_1 \quad F_2 \quad F_3 \quad F_4 \quad F_5]^T$$

The stepped bar is transformed into a finite element model using 1-D (line) elements.

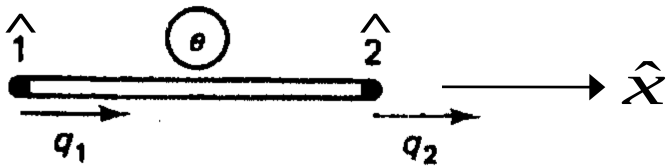


Global numbering

3-3-3 Element Connectivity

Consider a single line element. It lies in a local coordinate system, denoted by $\hat{x}$.

Note: Node number in local coordinate is denoted by a number with a hat on top.



Local numbering

q_1 and q_2 are nodal displacements in the local coordinate direction.

Element connectivity table

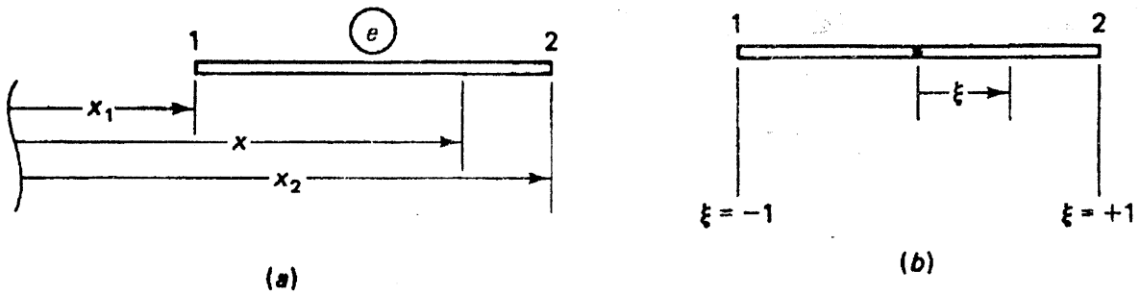
Elements	Nodes		
e	$\hat{1}$	$\hat{2}$	Local numbers
1	1	2	Global numbers
2	2	3	
3	3	4	
4	4	5	

Connectivity between global and local nodes must be established for each element, as tabulated in the table shown.

3.4 Natural Coordinate and Shape Functions

3-4-1 Natural Coordinate

Consider a single element. Local node 1 is at distance x_1 from a datum, and node 2 is at x_2, measured from the same datum point.



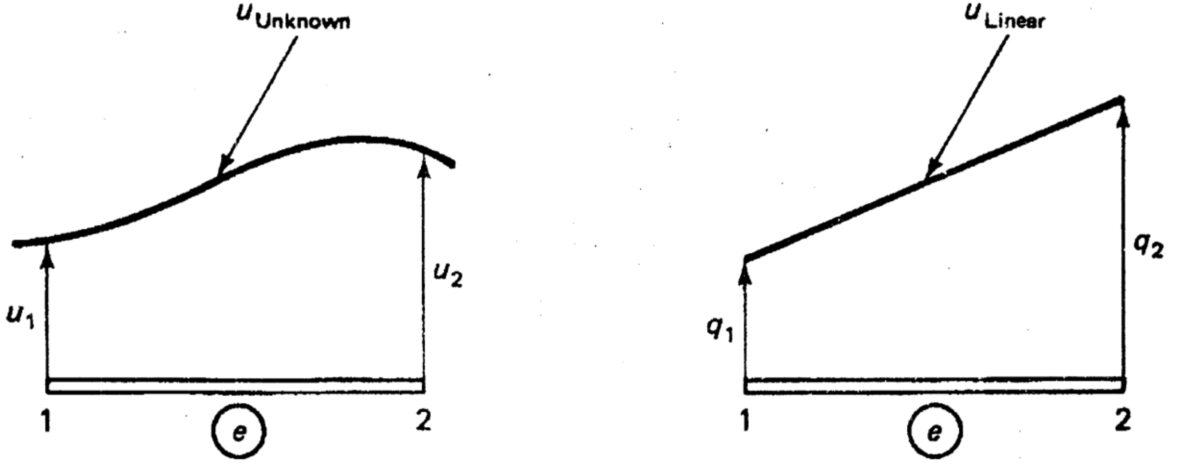
We define a natural or intrinsic coordinate system, ξ,

$$\xi = \frac{2}{(x_2 - x_1)}(x - x_1) - 1$$

Note: The ξ-coordinate will be used to define shape functions, required to establish interpolation function for the displacement field within the element.

3-4-2 Shape Functions

The displacement field, $u(x)$, within the element is not known. For simplicity, it is assumed that the displacement varies linearly from node 1 to node 2 within the element.



We establish a linear interpolation function to represent the linear displacement field within the element. To implement this, linear shape functions are defined, given by,

$$N_1(\xi) = \frac{1-\xi}{2} \quad \text{and} \quad N_2(\xi) = \frac{1+\xi}{2}$$

The linear displacement field, *u(x)*, within the element can now be expressed in terms of the linear shape functions and the local nodal displacement q_1 and q_2 as:

$$u(\hat{x}) = N_1 q_1 + N_2 q_2$$

$$u(\hat{x}) = \left(\frac{1-\xi}{2}\right) q_1 + \left(\frac{1+\xi}{2}\right) q_2$$

In matrix form:

$$u(\hat{x}) = [N]\{q\}$$

where $$[N] = [N_1 \quad N_2]$$

and $$\{q\} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [q_1 \quad q_2]^T$$

3-4-3 Isoparametric Formulation

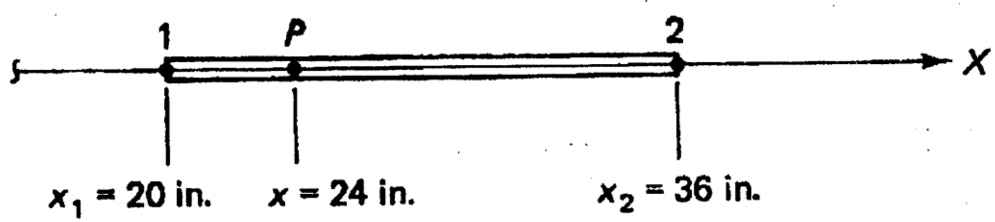
Coordinate x of any point on the element (measured from the same datum point as x_1 and x_2) can be expressed in terms of the same shape functions, N_1 and N_2 as

$$x = N_1 x_1 + N_2 x_2$$

$$x = \left(\frac{1-\xi}{2}\right) x_1 + \left(\frac{1+\xi}{2}\right) x_2$$

When the same shape functions N_1 and N_2 are used to establish interpolation function for coordinate of a point within an element and the displacement of that point, the formulation is specifically referred to as an isoparametric formulation.

Example 2-1



(a) Evaluate ξ, N_1, and N_2 at point P.

(b) If q_1 = 0.003 in and q_2 = -0.005 in, determine the value of displacement u at point P.

Solution

(a) The ξ coordinate of point *P* is given by

$$\xi_P = \frac{2}{x_2 - x_1}\left(x - x_1\right) - 1$$

$$= \frac{2}{36-20}\left(24-20\right) - 1$$

$$\xi_P = -0.5$$

The shape functions are:

$$N_1 = \frac{1-\xi}{2} = \frac{1+0.5}{2} = 0.75$$

$$N_2 = \frac{1+\xi}{2} = \frac{1-0.5}{2} = 0.25$$

(b) Displacement of point *P*

$$u_P = N_1 q_1 + N_2 q_2$$

$$= 0.75(0.003) + 0.25(-0.005)$$

$$u_P = 0.001 \text{ in}$$

3-5 Strain-Displacement Relation

Normal strain is related to displacement by

$$\varepsilon = \frac{du}{dx}$$

Using the chain rule of differentiation

$$\varepsilon = \frac{du}{d\xi}\frac{d\xi}{dx}$$

The two terms of the above relation are obtained as follows

$$\xi = \frac{2}{(x_2 - x_1)}(x - x_1) - 1 \quad \Rightarrow \quad \frac{d\xi}{dx} = \frac{2}{(x_2 - x_1)}$$

$$u = \left(\frac{1-\xi}{2}\right)q_1 + \left(\frac{1+\xi}{2}\right)q_2 \quad \Rightarrow \quad \frac{du}{d\xi} = \frac{(-q_1 + q_2)}{2}$$

Thus the normal strain relation can be written as

$$\varepsilon = \frac{1}{(x_2 - x_1)}\left[-q_1 + q_2\right]$$

which can be written in matrix form as

$$\varepsilon = [B]\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

where [*B*] is a row matrix called the <u>strain-displacement</u> matrix, given by

$$[B] = \frac{1}{(x_2 - x_1)}\begin{bmatrix} -1 & 1 \end{bmatrix} = \frac{1}{l_e}\begin{bmatrix} -1 & 1 \end{bmatrix}$$

since $x_2 - x_1$ = element length = l_e.

3-6 Stress-Strain Relation

Normal stress is related to the normal strain by a Hooke's law,

$$\sigma = E\varepsilon$$

where E is modulus of elasticity.

Substitute for the normal strain ε, we get,

$$\sigma = E\left[B\right]\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

Robert Hooke (1635-1703);
(Experimental Philosopher)

Theory of Minimum Potential Energy

3-7 Element Stiffness Matrix

We will use the potential energy approach to derive the element stiffness matrix [*k*] for the 1-D element.

Total potential energy of a body subjected to loads is given by,

$$\pi_p = U + \Omega$$

U = internal strain energy;
Ω = potential energy of external forces.

For the non-uniform bar, its total potential energy is given by

$$\pi_p = \frac{1}{2}\int_L \sigma^T \varepsilon A \, dx - \int_L u^T f A \, dx - \int_L u^T T \, dx - \sum_i Q_i P_i$$

Since the bar has been discretized into finite elements

$$\pi_p = \sum_e \frac{1}{2}\int_e \sigma^T \varepsilon A \, dx - \sum_e \int_e u^T f A \, dx - \sum_e \int_e u^T T \, dx - \sum_i Q_i P_i$$

We will derive the element stiffness matrix of the 1-D element using the internal strain energy term, U as follows,

$$U_e = \frac{1}{2}\int_e \sigma^T \varepsilon A\, dx$$

Recall, the stress and strain are given by

$$\sigma = E[B]\{q\} \quad \text{and} \quad \varepsilon = [B]\{q\}$$

Substitute these into the expression for U_e,

$$U_e = \frac{1}{2}\int_e E\left([B]\{q\}\right)^T [B]\{q\} A\, dx$$

$$= \frac{1}{2}\int_e \{q\}^T [B]^T E[B]\{q\} A\, dx$$

$$U_e = \frac{1}{2}\{q\}^T \left(\int_e [B]^T E[B] A\, dx\right)\{q\}$$

Recall again, $\dfrac{d\xi}{dx} = \dfrac{2}{l_e} \quad \Rightarrow \quad dx = \dfrac{l_e}{2} d\xi$

Substitute and simplifying the expression yields,

$$U_e = \frac{1}{2}\{q\}^T \left[[B]^T E_e [B] A_e \frac{l_e}{2} \int_{-1}^{+1} d\xi \right] \{q\}$$

$$= \frac{1}{2}\{q\}^T [B]^T E_e [B] A_e \frac{l_e}{2}(2)\{q\}$$

$$= \frac{1}{2}\{q\}^T A_e l_e E_e \frac{1}{l_e}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \frac{1}{l_e}\begin{bmatrix} -1 & 1 \end{bmatrix}\{q\}$$

$$= \frac{1}{2}\{q\}^T A_e l_e E_e \frac{1}{l_e^{\,2}}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}\begin{bmatrix} -1 & 1 \end{bmatrix}\{q\}$$

$$U_e = \frac{1}{2}\{q\}^T \frac{A_e E_e}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\{q\}$$

The internal strain energy for the 1-D element can now be written in the form,

$$U_e = \frac{1}{2}\{q\}^T [k]^e \{q\}$$

where $[k]^e$ represents the element stiffness matrix for the 1-D element, i.e.

$$[k]^e = \frac{E_e A_e}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Note: E_e = elastic modulus;
A_e = cross-sectional area;
l_e = element length.

3-8 Element Force Vector

The forces acting on 1-D structures can be of body force, f_b, traction force, T, and concentrated force, P. They may act individually in various combination.

The total potential energy of the structure,

$$\pi_p = \sum_e \frac{1}{2}\int_e \sigma^T \varepsilon A\, dx - \sum_e \int_e u^T f_b A\, dx - \sum_e \int_e u^T T\, dx - \sum_i Q_i P_i$$

a) Due to body force, f_b

The potential energy due to body force f_b in a single element is given by the second term, i.e.

$$\Omega_f = \int_e u^T f_b A\, dx$$

$$= \int_e \left(N_1 q_1 + N_2 q_2\right)^T f_b A\, dx$$

$$\Omega_f = A_e f_b \int_e \left(N_1 q_1 + N_2 q_2\right)^T dx$$

Rewrite,

$$\Omega_f = \{q\}^T \begin{Bmatrix} A_e f_b \int_e N_1 \, dx \\ A_e f_b \int_e N_2 \, dx \end{Bmatrix}$$

Recall that,

$$dx = \frac{l_e}{2} d\xi$$

Also,

$$\int_e N_1 \, dx = \frac{l_e}{2} \int_{-1}^{+1} \frac{1-\xi}{2} d\xi = \frac{l_e}{2}$$

$$\int_e N_2 \, dx = \frac{l_e}{2} \int_{-1}^{+1} \frac{1+\xi}{2} d\xi = \frac{l_e}{2}$$

- Show details of this integration.

Substitute and simplifying, yields

$$\Omega_f = \{q\}^T \begin{Bmatrix} \dfrac{A_e f_b l_e}{2} \\ \dfrac{A_e f_b l_e}{2} \end{Bmatrix} = \{q\}^T \cdot \frac{A_e l_e f_b}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

The potential energy due to the body force can now be expressed in the form,

$$\Omega_f = \{q\}^T \{f\}^e$$

where the force vector due to body force f_b is,

$$\{f\}^e = \frac{A_e l_e f_b}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Quiz: Can you give the physical interpretation of $\{f\}^e$?

b) Due to traction force, T

The potential energy due to traction force T is given by,

$$\Omega_T = \int_e u^T T \, dx = \int_e \left(N_1 q_1 + N_2 q_2 \right)^T T \, dx$$

Recall,

$$dx = \frac{l_e}{2} d\xi \qquad \int_e N_1 \, dx = \frac{l_e}{2} \int_{-1}^{+1} \frac{1-\xi}{2} d\xi = \frac{l_e}{2}$$

$$\int_e N_2 \, dx = \frac{l_e}{2} \int_{-1}^{+1} \frac{1+\xi}{2} d\xi = \frac{l_e}{2}$$

Rearranging and simplifying,

$$\Omega_T = \{q\}^T \begin{Bmatrix} T \int_e N_1 \, dx \\ T \int_e N_2 \, dx \end{Bmatrix} = \{q\}^T \begin{Bmatrix} \dfrac{l_e}{2} \\ \dfrac{l_e}{2} \end{Bmatrix}$$

The last equation is in the form,

$$\Omega_T = \{q\}^T \{T\}^e$$

i.e.

$$\Omega_T = \{q\}^T \cdot T \cdot \frac{l_e}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Thus, element traction force vector due to traction T,

$$\{T\}^e = \frac{T l_e}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Quiz: Can you give the physical interpretation of this?

Summary

We have established, for 1-D problems,

1. Stress-strain relation

$$\sigma = \frac{E}{l_e}\begin{bmatrix}-1 & 1\end{bmatrix}\begin{Bmatrix}q_1 \\ q_2\end{Bmatrix}$$

2. Element stiffness matrix

$$[k]^e = \frac{A_e E_e}{l_e}\begin{bmatrix}1 & -1 \\ -1 & 1\end{bmatrix}$$

3. Element force vector due to body force, f_b

$$\{f\}^e = \frac{A_e l_e f_b}{2}\begin{Bmatrix}1 \\ 1\end{Bmatrix}$$

4. Element force vector due to traction force, T

$$\{T\}^e = \frac{T l_e}{2}\begin{Bmatrix}1 \\ 1\end{Bmatrix}$$

Example 3-2

A thin steel plate has a uniform thickness t = 1 in., as shown. Its elastic modulus, E = 30 x 10^6 psi, and weight density, ρ = 0.2836 lb/in^3.

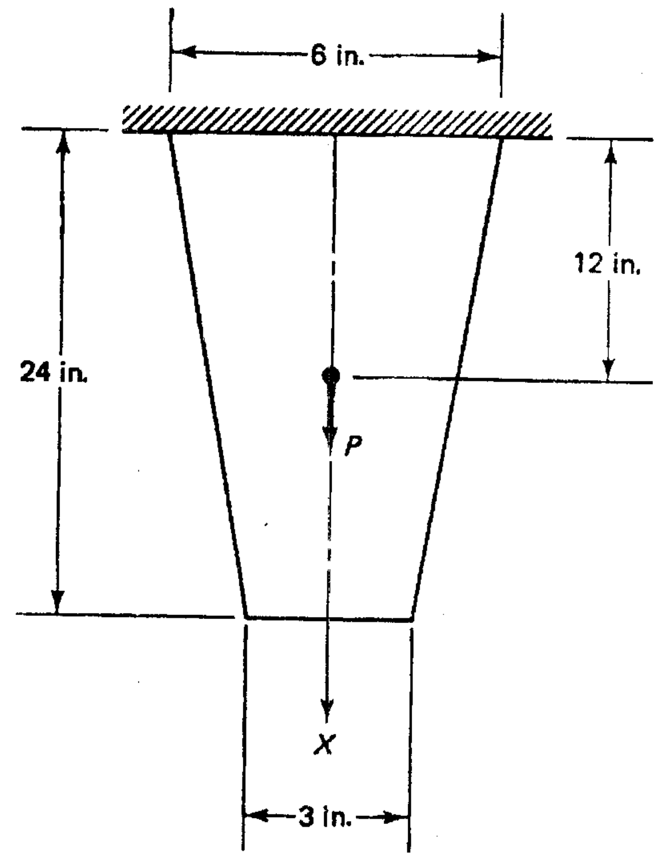
The plate is subjected to a point load P = 100 lb at its midpoint and a traction force T = 36 lb/ft.

Determine:

a) Displacements at the mid-point and at the free end,

b) Normal stresses in the plate, and

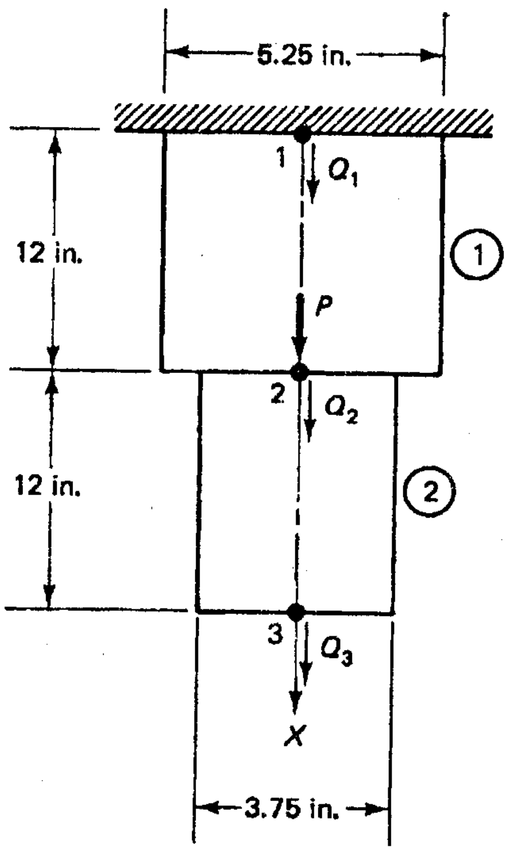
c) Reaction force at the support.



(a)

Solution

1. Transform the given plate into 2 sections, each having uniform cross-sectional area.



(b)

Note:

Area at midpoint is

A_{mid} = 4.5 in^2.

Average area of section 1 is

A_1 = (6 + 4.5)/2 = 5.25 in^2.

Average area of section 2 is

A_2 = (4.5 + 3)/2 = 3.75 in^2.

2. Model each section using 1-D (line) element.

3. Write the element stiffness matrix for each element

element 1:

$$[k]^{(1)} = \frac{5.25\times 30\times 10^6}{12}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

element 2:

$$[k]^{(2)} = \frac{3.75\times 30\times 10^6}{12}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

4. Assemble global stiffness matrix,

$$[K] = \frac{30\times 10^6}{12}\begin{bmatrix} 5.25 & -5.25 & 0 \\ -5.25 & 9.00 & -3.75 \\ 0 & -3.75 & 3.75 \end{bmatrix}$$

Note: The main diagonal must contain positive numbers only!

5. Write the element force vector for each element

a) Due to body force, f_b = 0.2836 lb/in^3

element 1

$$\{f_b\}^{(1)} = \frac{5.25 \times 12 \times 0.2836}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

element 2

$$\{f_b\}^{(2)} = \frac{3.75 \times 12 \times 0.2836}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Assemble global force vector due to body force,

$$\{F_b\} = \frac{12 \times 0.2836}{2} \begin{Bmatrix} 5.25 \\ 9.00 \\ 3.75 \end{Bmatrix} = \begin{Bmatrix} 8.9 \\ 15.3 \\ 6.4 \end{Bmatrix}$$

b) Due to traction force, T = 36 lb/ft

element 1

$$\{T\}^{(1)} = \frac{\left(\frac{36}{12}\right)\times 12}{2}\begin{Bmatrix}1\\1\end{Bmatrix} = 18\begin{Bmatrix}1\\1\end{Bmatrix}$$

element 2

$$\{T\}^{(2)} = \frac{\left(\frac{36}{12}\right)\times 12}{2}\begin{Bmatrix}1\\1\end{Bmatrix} = 18\begin{Bmatrix}1\\1\end{Bmatrix}$$

Assemble global force vector due to traction force,

$$\{F_T\} = 18\begin{Bmatrix}1\\2\\1\end{Bmatrix} = \begin{Bmatrix}18\\36\\18\end{Bmatrix}$$

c) Due to concentrated load, P = 100 lb at node 2

$$\{F_P\} = \begin{Bmatrix} 0 \\ 100 \\ 0 \end{Bmatrix}$$

6. Assemble all element force vectors to form the global force vector for the entire structure.

$$\{F\} = \begin{Bmatrix} 8.9+18+0 \\ 15.3+36+100 \\ 6.4+18+0 \end{Bmatrix} = \begin{Bmatrix} 26.9 \\ 151.3 \\ 24.4 \end{Bmatrix} \quad lb$$

7. Write system of linear equations (SLEs) for entire model

The SLEs can be written in condensed matrix form as

$$[K]\{Q\}=\{F\}$$

Expanding all terms and substituting values, we get

$$\frac{30\times10^{6}}{12}\begin{bmatrix}5.25 & -5.25 & 0\\ -5.25 & 9.00 & -3.75\\ 0 & -3.75 & 3.75\end{bmatrix}\begin{Bmatrix}Q_1\\ Q_2\\ Q_3\end{Bmatrix}=\begin{Bmatrix}26.9\\ 151.3\\ 24.4\end{Bmatrix}$$

Note:

1. The global force term includes the unknown reaction force R_1 at the support. But it is ignored for now.
2. The SLEs have no solutions since the determinant of $[K] = 0$; Physically, the structure moves around as a rigid body.

8. Impose boundary conditions (BCs) on the global SLEs

There are 2 types of BCs:

a) Homogeneous = specified zero displacement;

b) Non-homogeneous = specified non-zero displacement.

In this example, homogeneous BC exists at **node 1**. How to impose this BC on the global SLEs?

DELETE ROW AND COLUMN #1 OF THE SLEs!

$$\frac{30\times10^6}{12}\begin{bmatrix} 5.25 & -5.25 & 0 \\ -5.25 & 9.00 & -3.75 \\ 0 & -3.75 & 3.75 \end{bmatrix}\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = \begin{Bmatrix} 26.9 \\ 151.3 \\ 24.4 \end{Bmatrix}$$

9. Solve the reduced SLEs for the unknown nodal displacements

The reduced SLEs are,

$$\frac{30\times10^{6}}{12}\begin{bmatrix} 9.00 & -3.75 \\ -3.75 & 3.75 \end{bmatrix}\begin{Bmatrix} Q_2 \\ Q_3 \end{Bmatrix} = \begin{Bmatrix} 151.3 \\ 24.4 \end{Bmatrix}$$

Solve using Gaussian elimination method, yields

$$\begin{Bmatrix} Q_2 \\ Q_3 \end{Bmatrix} = \begin{Bmatrix} 1.339\times10^{-5} \\ 1.599\times10^{-5} \end{Bmatrix} \quad in$$

Quiz: Does the answers make sense? Explain…

10. Estimate stresses in each elements

Recall, $$\sigma^{(e)} = E[B]\{q\} = E \cdot \frac{1}{l_e}\begin{bmatrix} -1 & 1 \end{bmatrix}\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

element 1

$$\sigma^{(1)} = 30 \times 10^6 \cdot \frac{1}{12}\begin{bmatrix} -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ 1.339 \times 10^{-5} \end{Bmatrix} = 33.48 \text{ psi}$$

element 2

$$\sigma^{(2)} = 30 \times 10^6 \cdot \frac{1}{12}\begin{bmatrix} -1 & 1 \end{bmatrix}\begin{Bmatrix} 1.339 \times 10^{-5} \\ 1.599 \times 10^{-5} \end{Bmatrix} = 6.5 \text{ psi}$$

11. Compute the reaction force R_1 at node 1

We now include the reaction force term in the global SLEs. From the 1st. equation we get,

$$\frac{30\times10^6}{12}\begin{bmatrix} 5.25 & -5.25 & 0 \\ -5.25 & 9.00 & -3.75 \\ 0 & -3.75 & 3.75 \end{bmatrix}\begin{Bmatrix} 0 \\ 1.339\times10^{-5} \\ 1.599\times10^{-5} \end{Bmatrix} = \begin{Bmatrix} 26.9 + R_1 \\ 151.3 \\ 24.4 \end{Bmatrix}$$

We have,

$$R_1 = \frac{30\times10^6}{12}\begin{bmatrix} 5.25 & -5.25 & 0 \end{bmatrix}\begin{Bmatrix} 0 \\ 1.339\times10^{-5} \\ 1.599\times10^{-5} \end{Bmatrix} - 26.9334$$

$$R_1 = -202.68 \text{ lb}$$

Example 3-3

A concentrated load P = 60 kN is applied at the midpoint of a uniform bar as shown.

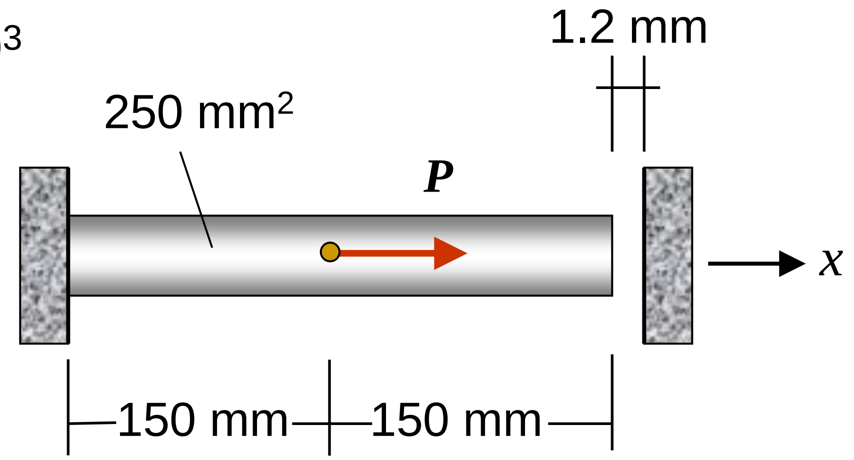
Initially, a gap of 1.2 mm exists between the right end of the bar and the support there.

If the elastic modulus $E = 20 \times 10^3$ N/mm^2, determine the:

a) displacements field,

b) stresses in the bar, and

c) reaction force at the support.



Solution

1. Write the element stiffness matrices and assemble the global stiffness matrix.

$$[K] = \frac{20\times10^3\times250}{150}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

2. Write the element force vectors and assemble the global force vector.

$$\{F\} = \begin{bmatrix} 0, & 60\times10^3, & 0 \end{bmatrix}^T$$

3. Write the global system of linear equations.

$$\frac{10^3}{15}\begin{bmatrix} 500 & -500 & 0 \\ -500 & 1000 & -500 \\ 0 & -500 & 500 \end{bmatrix}\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = 10^3\begin{Bmatrix} 0 \\ 60 \\ 0 \end{Bmatrix}$$

4. Impose the boundary conditions.

We have; $Q_1 = 0$; $Q_3 = 1.2$ mm. Using Gaussian elimination method:

a) Delete 1st row and column.

b) Delete 3rd row and column and modify the force term.

$$\frac{10^3}{15}\begin{bmatrix} 500 & -500 & 0 \\ -500 & 1000 & -500 \\ 0 & -500 & 500 \end{bmatrix}\begin{Bmatrix} Q_1 \\ Q_2 \\ 1.2 \end{Bmatrix} = 10^3\begin{Bmatrix} 0 \\ 60 \\ 0 \end{Bmatrix}$$

The reduced SLE becomes,

$$\frac{10^3}{15}[1000]\{Q_2\} = 10^3\left\{60 + \frac{500(1.2)}{15}\right\}$$

Modification to force term

7. Solve the reduced SLE, we get

$$Q_2 = 1.5 \text{ mm}$$

8. Compute stresses in the bar,

$$\sigma_1 = 20 \times 10^3 \times \frac{1}{150} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 1.5 \end{Bmatrix}$$

$$\sigma_1 = 200 \text{ MPa}$$

$$\sigma_2 = 20 \times 10^3 \times \frac{1}{150} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} 1.5 \\ 1.2 \end{Bmatrix}$$

$$\sigma_2 = -40 \text{ MPa}$$

9. Compute reaction forces at supports

Using the 1st and 3rd equations, we obtain,

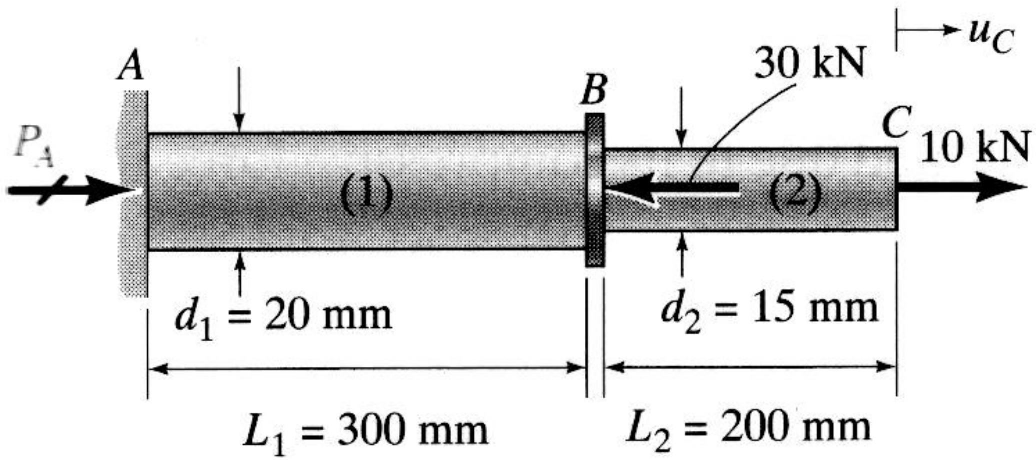
R_1 = -50 x 10^3 N; R_3 = -10 x 10^3 N.

Exercise 2-1

A composite bar ABC is subjected to axial forces as shown. Given, the elastic moduli, E_1 = 200 GPa and E_2 = 70 GPa. Estimate:

a) Displacement of end C; [Answer: δ_C = 6.62x10^{-2} mm]
b) Stress in section 2, and
c) Reaction force at support A.

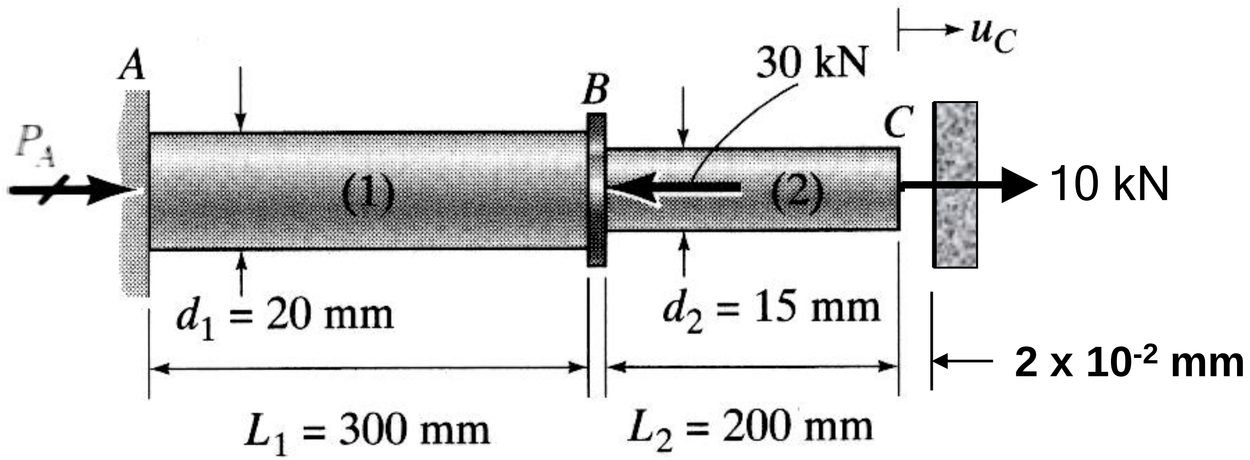
Verify your results with analytical solution.



Exercise 2-2

Reconsider Exercise 2-1. Suppose a gap of $\delta = 2 \times 10^{-2}$ mm exists between end *C* and a fixed support there. Estimate:

a) Displacement of point *B*;
b) Stress in section 1, and
c) Reaction forces at both supports.



Assignment 2-1

- Find a journal paper on the application of finite element method to model and simulate real world problems, from various journals on the internet.
  (e.g. : www.sciencedirect.com).
- Download the paper (in PDF format), and print it.
- Read the paper and make one (1) page summary on the content of the paper - typewritten.
- Submit the summary and copy of the paper to me. Use cover page.
- Due in: 7 days time.