KINEMATICS OF FLUID MOTION

The Velocity Field

The representation of properties of fluid parameters as function of the spatial coordinates is termed a field representation of the flow.

One of the most important fluid variables is the velocity field.

\[ V = u(x, y, z, t) \hat{i} + v(x, y, z, t) \hat{j} + w(x, y, z, t) \hat{k} \]

Where \( u, v \) and \( w \) are the \( x, y \) and \( z \) components of the velocity vector.

The speed of fluid is:

\[ V = |V| = \sqrt{u^2 + v^2 + w^2} \]
Example 1

The velocity field of a flow is given by
\[ V = (5z - 3)\hat{i} + (x + 4)\hat{j} + (4y)\hat{k} \] (m/s)

Determine the fluid speed at the origin \((x=y=z=0)\)
And on the \(x\)-axis \((y=z=0)\)

Example 2

The velocity field of a flow is given by
\[ V = \left( \frac{20y}{\sqrt{x^2 + y^2}} \right)\hat{i} - \left( \frac{20x}{\sqrt{x^2 + y^2}} \right)\hat{j} \] (m/s)

Determine the fluid speed at points along the \(x\)-axis and along the \(y\)-axis.
What is the angle between the velocity vector and the \(x\)-axis at point \((x,y) = (5,0), (5,5)\) and \((0,5)\)

Example 3

The components of a velocity vector field are given by \(u=x+y, v=xy^3+16\) and \(w=0\).
Determine the location of any stagnation points \((V=0)\) in the flow field.
Eulerian and Lagrangian Flow Descriptions

There are two general approaches in analyzing fluid mechanics problems.

The first method is called the Eulerian method, the second method is called the Lagragian method.

From Eulerian method we obtain information about the flow in terms of what happens at fixed points in space as the fluid flows past those points.

Lagrangian method involves following individual fluid particles as they move about and determining how the fluid properties associated with these particles change as a function of time. That is, the fluid particles are “tagged” or identified, and their properties determined as they move.
Steady and Unsteady Flow

We have assumed steady flow is the velocity at a given point in space does not vary with time.

\[
\frac{\partial V}{\partial t} = 0
\]

In reality, almost all flows are unsteady in some sense.

The unsteady flows are usually more difficult to analyze and to investigate experimentally than are steady flows.

Laminar flow – smooth flow

Turbulent flow – irregular flow
Streamlines

Streamline is a line that is everywhere tangent to the velocity field.

Streamlines are obtained analytically by integrating the equations defining lines tangent to the velocity field as illustrated in the Figure 3.

\[ \frac{dy}{dx} = \frac{v}{u} \]
Streaklines and pathlines

A streakline consists of all particles in a flow that have previously passed through a common point.

Streaklines are more of a laboratory tool than an analytical tool. They can be obtained by taking instantaneous photographs of marked particles that all passed through a given location in the flow field at some earlier time.

A pathline is the line traced out by a given particle as it flows from one point to another. The pathline is a Lagrangian concept that can be produced in the laboratory by marking a fluid particle and taking a time exposure photograph of its motion.

Figure 4
The Acceleration Field

For the infrequently used Lagrangian method, we described the fluid acceleration just as is done in solid body dynamics, \( a = a(t) \) for each particle.

For the Eulerian description we describe the acceleration field as a function of position and time without actually following any particular particle.

This is analogous to describing the flow in terms of the velocity field,

\[
V = V(x, y, z, t)
\]

The acceleration of a particle is the time rate of change of its velocity.
The Material Derivative

Consider a fluid particle moving along its pathline as shown in Figure 5.

In general, the particle’s velocity, denoted $V_A$ for particle A, is a function of its location and the time. That is:

$$V_A = V_A(r_A, t) = V_A[x_A(t), y_A(t), z_A(t), t]$$
Then, the acceleration field from the velocity field for any particle obtained as:

\[
a = \frac{\partial V}{\partial t} + u \frac{\partial V}{\partial x} + v \frac{\partial V}{\partial y} + w \frac{\partial V}{\partial z}
\]

This is a result whose scalar components can be written as:

(x-axis)

\[
a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}
\]

(y-axis)

\[
a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}
\]

(z-axis)

\[
a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}
\]
Convective Effects

The portion of the material derivative represented by the spatial derivatives is termed the convective derivative.

It represents the fact that a flow property associated with a fluid particle may vary because of the motion of the particle from one point in space where the parameter has one value to another point in space where its value is different.

That portion of the acceleration is termed the convective acceleration.

For example, the temperature of a water particle changes as it flows through a water heater.

Figure 6
Example 4
The x and y components of a velocity field are given by \( u=x^2y \) and \( v=-xy^2 \). Determine the equation for the streamlines of this flow.

Example 5
A velocity field is given by \( u=cx^2 \) and \( v=cy^2 \), where \( c \) is a constant. Determine the \( x \) and \( y \) components of the acceleration. At what point (points) in the flow is the acceleration zero.

Example 6
Determine the acceleration field for a three-dimensional flow with velocity components, \( u=-x \), \( v=4x^2y^2 \) and \( w=x-y \).
Example 7

A hydraulic jump is a rather sudden change in depth of a liquid layer as it flows in an open channel as shown in Figure 7. If \( V_1 = 0.4 \text{ m/s} \), \( V_2 = 0.1 \text{ m/s} \) and \( \ell = 0.07 \text{ m} \), estimate the average deceleration of the liquid as it flows across the hydraulic jump. How many G’s deceleration does this represent?

Example 8

\( V_0 = 40 \text{ m/s} \) and \( V = \frac{3}{2} V_0 \sin \theta \). Determine the streamwise and normal components of acceleration at point \( A \) if the radius of the sphere is \( a = 0.20 \text{ m} \).
Streamline Coordinates

In many flow situations, it is convenient to use a coordinate system defined in terms of the streamlines of the flow.

The flows can be described either in $(x, y)$ Cartesian coordinate or $(r, \theta)$ polar coordinate system.
A control volume, on the other hand, is a volume in space (a geometric entity, independent of mass) through which fluid may flow.

A system is a specific, identifiable quantity of matter. It may consist of a relatively large amount of mass, or it may be an infinitesimal size.

The system may interact with its surroundings by various means. It may continually change size and shape, but it always contains the same mass.
KINEMATICS OF FLUID MOTION

Reynolds Transport Theorem

We are sometimes interested in what happens to a particular part of the fluid as it moves about.

Other times we may be interested in what effect the fluid has on a particular object or volume in space as fluid interact with it.

Thus, we need to describe the laws governing fluid motion using both system concept (consider a given mass of the fluid) and control volume concept (consider a given volume).

To do this we need an analytical tool to shift from one representation to the other.

The *Reynolds Transport Theorem* provides this tool.
All physical laws are stated in terms of various physical parameters.

Velocity, acceleration, mass, temperature and momentum are but a few of the more common parameters.

Let \( B \) represent any of these (or other) fluid parameters and \( b \) represent the amount of that parameter per unit mass. That is;

\[
B = m \cdot b
\]

where \( m \) is the mass of the portion of fluid of interest.

The \( B \) is termed an extensive property and \( b \) is termed an intensive property.

If \( B \) is mass, it follows that \( b \) is equal to 1.

If \( B \) is the kinetic energy of mass, \( B = m \frac{V^2}{2} \), then \( b = \frac{V^2}{2} \), the kinetic energy per mass.
General form of the Reynolds transport theorem for a fixed, non-deforming control volume is given as:

\[
\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho b dV + \int_{cs} \rho b V \cdot \hat{n} \cdot dA
\]
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The velocity field of a flow is given by
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Determine the fluid speed at points along the x-axis and along the y-axis.
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Example 3

The components of a velocity vector field are given by \(u=x+y, v=xy^3+16\) and \(w=0\).
Determine the location of any stagnation points \((V=0)\) in the flow field.
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The x and y components of a velocity field are given by \( u = x^2 y \) and \( v = -xy^2 \). Determine the equation for the streamlines of this flow.

Example 5
A velocity field is given by \( u = cx^2 \) and \( v = cy^2 \), where \( c \) is a constant. Determine the x and y components of the acceleration. At what point (points) in the flow is the acceleration zero.

Example 6
Determine the acceleration field for a three-dimensional flow with velocity components, \( u = -x \), \( v = 4x^2 y^2 \) and \( w = x - y \).

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Example 9

Water flows through the 2-m-wide rectangular channel shown in Figure 9 with a uniform velocity 3 m/s.

(a) Directly integrate Eq. 4.16 with \( b=1 \) to determine the mass flowrate (kg/s) across section CD of the control volume.

(b) Repeat part (a) with \( b=1/\rho \), where \( \rho \) is the density.

Explain the physical interpretation of the answer to part (b).
The wind blows across a field with an approximate velocity profile as shown in Figure 11. Use Eq. 4.16 with the parameter $b$ equal to the velocity to determine the momentum flowrate across the vertical surface $A$-$B$, which is of unit depth into the paper.