HYDROSTATIC FORCE ON A SUBMERGED PLANE SURFACE

When a surface is submerged in a fluid, forces develop on the surface due to the fluid.

The determination of these forces is important in the design of storage tanks, ships, dams, and other hydraulic structures.

For fluids at rest we know that the force must be \textit{perpendicular} to the surface since there are no shearing stresses present.

We also know that the pressure will vary linearly with depth as shown in Figure 1 if the fluid is incompressible.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Pressure on tank bottom (a) and pressure on tank ends (b).}
\end{figure}
The magnitude of the resultant fluid force is equal to the pressure acting at the centroid of the area multiplied by the total area

\[ F_R = \rho gh_c \cdot A \]  

(equation 1)
Note that the magnitude of the force is independent of the angle $\theta$ and depends only on the specific weight of the fluid, the total area, and the depth of the centroid of the area below the surface.

In effect, Equation 1 indicates that the magnitude of the resultant force is equal to the pressure at the centroid of the area multiplied by the total area.

Since all the differential forces that were summed to obtain $F_R$ are perpendicular to the surface, the resultant $F_R$ must also be perpendicular to the surface.

The point through which the resultant force acts is called the center of pressure.
Coordinate for center of pressure \((y_R, x_R)\):

\[
y_R = \frac{I_{xc}}{y_c A} + y_c
\]

\[
x_R = \frac{I_{xy_c}}{y_c A} + x_c
\]
Centroidal coordinates and moments of inertia for some common areas are given in Figure 3.

(a) Rectangle

\[ A = ba \]
\[ I_x = \frac{1}{12} ba^3 \]
\[ I_y = \frac{1}{12} ab^3 \]
\[ I_{xy} = 0 \]

(b) Circle

\[ A = \pi R^2 \]
\[ I_x = I_y = \frac{\pi R^4}{4} \]
\[ I_{xy} = 0 \]

(c) Semicircle

\[ A = \frac{\pi R^2}{2} \]
\[ I_x = 0.1098R^4 \]
\[ I_y = 0.3927R^4 \]
\[ I_{xy} = 0 \]

(d) Triangle

\[ A = \frac{ab}{2} \]
\[ I_x = \frac{bR^3}{36} \]
\[ I_y = \frac{bR^2}{12} (b - 2d) \]
\[ I_{xy} = 0 \]

(e) Quarter circle

\[ A = \frac{\pi R^2}{4} \]
\[ I_x = I_y = 0.05488R^4 \]
\[ I_{xy} = -0.01647R^4 \]

Figure 3
Chapter 3 – Pressure prism for rectangular shape

**PRESSURE PRISM**

An informative and useful graphical interpretation can be made for the force developed by a fluid acting on a plane area.

Consider the pressure distribution along a vertical wall of a tank of width $b$, which contains a liquid having a specific weight $\gamma(=\rho g)$.

Since the pressure must vary linearly with depth, we can represent the variation as is shown in Figure 4, where the pressure is equal to zero at the upper surface and equal to $\gamma h(=\rho gh)$ at the bottom.

![Figure 4](image_url)
The base of this “volume” in pressure-area space is the plane surface of interest, and its altitude at each point is the pressure.

This volume is called the pressure prism, and it is clear that the magnitude of the resultant force acting on the surface is equal to the volume of the pressure prism.

“The magnitude of the resultant fluid force is equal to the volume of the pressure prism and passes through its centroid”
Specific values can be obtained by decomposing the pressure prism into two parts, $ABDE$ and $BCD$, as shown in Figure 5. Thus,

$$F_R = F_1 + F_2$$

$$F_R = \text{volume} = \frac{1}{2}(\rho gh)(bh) = \rho g \left(\frac{h}{2}\right)A$$

The location of $F_R$ can be determined by summing moments about some convenient axis, such as one passing through $A$. In this instance

$$F_R y_A = F_1 y_1 + F_2 y_2$$

Figure 5
For inclined plane surfaces the pressure prism can still be developed, and the cross section of the prism will generally be trapezoidal as is shown in Figure 6.

The use of pressure prisms for determining the force on submerged plane areas is convenient if the area is rectangular so the volume and centroid can be easily determined.
However, for other nonrectangular shapes, integration would generally be needed to determine the volume and centroid.

In these circumstances it is more convenient to use the equations developed in the previous section, in which the necessary integrations have been made and the results presented in a convenient and compact form that is applicable to submerged plane areas of any shape.
We note that in this case the force on one side of the wall now consists of $F_R$ as a result of the hydrostatic pressure distribution, plus the contribution of the atmospheric pressure, $p_{\text{atm}}A$, where $A$ is the area of the surface.

However, if we are going to include the effect of atmospheric pressure on one side of the wall we must realize that this same pressure acts on the outside surface (assuming it is exposed to the atmosphere), so that an equal and opposite force will be developed as illustrated in the figure 7.

Thus, we conclude that the *resultant* fluid force on the surface is that due only to the gage pressure contribution of the liquid in contact with the surface—the atmospheric pressure does not contribute to this resultant.
Question 1

A 3-m-wide, 8-m-high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in Figure 1. The gate is hinged at its bottom and held closed by a horizontal force, $F_H$, located at the center of the gate. The maximum value for $F_H$ is 3500 kN.

(a) Determine the maximum water depth, $h$, above the center of the gate that can exist without the gate opening.

(b) Is the answer the same if the gate is hinged at the top? Explain your answer.
Question 2

An open tank has a vertical partition and on one side contains gasoline with a density $\rho = 700 \text{ kg/m}^3$ at a depth of 4 m, as shown in Figure 2. A rectangular gate that is 4 m high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, $h$, will the gate start to open?

Figure 2
Question 3

A rectangular gate that is 2 m wide is located in the vertical wall of a tank containing water as shown in Figure 3. It is desired to have the gate open automatically when the depth of water above the top of the gate reaches 10 m.

(a) At what distance, $d$, should the frictionless horizontal shaft be located?

(b) What is the magnitude of the force on the gate when it opens?

Figure 3
Question 4

A rectangular gate having a width of 5 m is located in the sloping side of a tank as shown in Figure 4. The gate is hinged along its top edge and is held in position by the force $P$. Friction at the hinge and the weight of the gate can be neglected. Determine the required value of $P$. 

Figure 4
Question 5

A homogeneous, 4-m-wide, 8-m-long rectangular gate weighing 300 kg is held in place by a horizontal flexible cable as shown in Figure 5. Water acts against the gate which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.
Question 6

A vertical plane area having the shape shown in Figure 6 is immersed in an oil bath (specific weight = 8.75 kN/m³). Determine the magnitude of the resultant force acting on one side of the area as a result of the oil.

Figure 6
Question 7

A 90-kg homogeneous gate of 10-m width and 5-m length is hinged at point $A$ and held in place by a 12-m-long brace as shown in Figure 7. As the bottom of the brace is moved to the right, the water level remains at the top of the gate. The line of action of the force that the brace exerts on the gate is along the brace.

(a) Plot the magnitude of the force exerted on the gate by the brace as a function of the angle of the gate, $\theta$, for $0 \leq \theta \leq 90^\circ$.

(b) Repeat the calculations for the case in which the weight of the gate is negligible. Comment on the results as $\theta \to 0$.

![Figure 7](image)

**Answer:**

1. (a) $h = 16.2$ m  (b) $h = 13.5$ m
2. $h = 3.55$ m
3. (a) $d = 2.11$ m  (b) $F_R = 941.8$ kN
4. $P = 1.78$ MN
5. $T = 177$ kN
6. $F_R = 373$ kN
PAST YEAR QUESTION FOR
CHAPTER 2: HYDROSTATIC FORCE ON A SUBMERGED PLANE SURFACE

Question 1

![Diagram showing hydrostatic forces on a submerged plane surface]

a. Apakah tujah hidrostatik dan nyatakan tiga (3) faktor yang mempengaruhi magnitude tujah ini.

b. Sebuah pintu segiempat tepat dengan lebar 0.5m, di engsel seperti yang ditunjukkan dalam Rajah 1. Pintu ini digunakan untuk memisahkan dua buah tangki air. Pintu ini mula terbuka apabila aras air di dalam tangki sebelah kiri turun sebanyak 0.5m dan aras air di dalam tangki sebelah kanan kekal pada ketinggian 2m. Tentukan berat pintu.
Question 2

Pintu segi empat tepat nipis (berat 10kN) dengan ketinggian 5m dan lebar 2m digunakan untuk memisahkan dua jenis bendalir. Pintu sepatutnya berada di dalam keadaan tegak. Walau bagaimanapun, system kawalan bendalir rosak menyebabkan air melebihi ketinggian yang sepatutnya. Akibat daripada kerosakan itu, pintu telah condong seperti yang dilakarkan dalam Rajah 2. Untuk memastikan pintu tidak jatuh, daya dengan magnitude 24kN dikenakan dihujung pintu. Dengan data yang diberikan, tentukan kedalaman $H$ bendalir $B$ yang berjaya menampung pintu supaya ia tidak terus jatuh.
Question 3

a. Dengan bantuan gambarajah, takrifkan pusat tekanan (CP: Center of Pressure) dan pusat sentroid, C bagi sebuah plat rata tegak yang tenggelam di dalam air static.

b. Dua buah tangki air dipisahkan dengan sebuah pintu segiempat tepat yang mempunyai panjang 3m dan lebar 1.7m seperti yang ditunjukkan dalam Rajah 3. Pintu dengan jisim 900kg itu di engsel pada titik O dengan geseran pada engsel itu boleh diabaikan. Jika aras air adalah seperti yang ditunjukkan dalam rajah, tentukan ketinggian $H$ yang maksimum bagi memastikan pintu itu masih tertutup.
The gate shown in Figure 4 is 2m wide and hinged at point A. Calculate the force required at point B to open the gate if the mass of the gate is 10kg.

**Answer :**

1. $F_R = 10.61 \text{ kN}, Y_{FR} = 1.62 \text{ m}, F_L = 2.82 \text{ kN}, Y_{FL} = 0.77 \text{ m}, \text{weight of gate} = 16.7 \text{ kN}$
2. $F_R = 101.8 \text{ kN}, Y_{FR} = 2.31 \text{ m}, F_L = 14.27H^2 \text{ kN}, Y_{FL} = 0.77H \text{ m}, \text{weight of gate} = 16.7 \text{ kN}$
3. $F_A = (50031H + 57536) \text{ N}, Y_{FA} = \frac{0.75}{(1.31H + 1.5)} + (1.31H + 1.5) \text{ m}, F_B = 117.6 \text{ kN}$

   $Y_{FB} = 3.31 \text{ m}, H^2 - 0.1556H - 1.5 = 0, H = 1.305 \text{ m}$
4. $F_R = 161865 \text{ N}, Y_R = 5.64 \text{ m}, B = 88.5 \text{ kN}$