Application of memetic algorithm in modelling discrete-time multivariable dynamics systems

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Abstract

Evolutionary algorithm (EA) such as genetic algorithm (GA) has demonstrated to be an effective method for identification of single-input–single-output (SISO) system. However, for multivariable systems, increasing the orders and the non-linear degrees of the model will result in excessively complex model and the identification procedure for the systems is more often difficult because couplings between inputs and outputs. There are more possible structures to choose from and more parameters are required to obtain a good fit. In this work, a new model structure selection in system identification problems based on a modified GA with an element of local search known as memetic algorithm (MA) is adopted. This paper describes the procedure and investigates the performance and the effectiveness of MA based on a few case studies. The results indicate that the proposed algorithm is able to select the model structure of a system successfully. A comparison of MA with other algorithms such as GAs demonstrates that MA is capable of producing adequate and parsimonious models effectively.

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1. Introduction

System identification is a process of developing an accurate model of a dynamic system \cite{1,2}. Most researchers in system identification focus on two main areas: the selection of model structure that consists of adequate number of terms in the final model and the estimation of the parameter of those terms. For multivariable systems, increasing the order and the degree of non-linearity of the model will result in excessively complex model and the identification procedure for the systems are more often difficult because couplings between inputs and outputs \cite{3}. There are some examples of different types of model structure representations used for multivariable system identification such as state-space model \cite{4,5} and radial-basis neural network model \cite{6}. For parameter estimation, there are also various methods used such as stochastic

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identification algorithm [4], prediction error method [7], orthogonal least-square method [7] and iterative multistage estimation algorithm [8].

The main problem for identification of multi-input–multi-output (MIMO) systems is the problem of dimensionality where the polynomial structure of the model increase drastically as the number of input and output variables of the system increases [8]. Therefore, the important step for developing an algorithm for system identification in a MIMO process is the development of an efficient mean of model structure selection. An alternative approach for determining model structures for MIMO system is to implement evolutionary algorithm (EA) such as genetic algorithm. Recently, EA such as genetic algorithm or GA has been proposed which attempt to alleviate the problems associated with gradient methods. GA has been applied in many applications [9–12] and has demonstrated to be an effective method of system identification of single-input–single-output (SISO) system. Some related works on GA applications to system identification have been published [13–15]. Most of those methods were applied to the identification of SISO non-linear systems and can be extended to MIMO system. There were some applications of GA to MIMO system identification; however, GA was used to estimate the parameters of the models [16–18]. It was pointed out in the literature [19] that the search ability of GA to find global solution is a bit inferior to local search. Whitley et al. [20] demonstrated that by applying local search in GA for some multi-modal problems, the performance improved substantially. Thus, hybrid GA with local search technique is proposed to improve the search ability in genetic algorithm. Recognising the similarities with other population-based approaches but having the advantages of local search mechanism, Moscato [21] introduced memetic algorithm (MA), an EA incorporating local search technique and it became widely accepted and has shown promising results in many applications [21–24].

This paper presents the identification of multivariable systems by incorporating modified GA and local search algorithm, which is called memetic algorithm or MA. To validate the algorithm, this study starts with the identification of simulated MIMO systems with known model structures to show that the algorithm is capable of identifying them effectively. Later, a case study is considered where the inputs and outputs for this multivariable system was obtained from real experimental data. The process considered is a perfectly mixed, jacketed continuously stirred tank reactor or CSTR [25].

This paper is organised as follows. Section 2 reviews multivariable system and polynomial Non-linear AutoRegressive with eXogeneous inputs (NARX) model representation of a system. Section 3 describes the algorithm used for model structure selection while Section 4 presents and discusses simulation study conducted to identify a MIMO model of the CSTR system based on a real experimental data. Section 5 summarises the main contribution of the paper.

2. Multivariable systems

Systems with several inputs and/or outputs are called multivariable systems. These systems are more often difficult to model because the couplings between inputs and outputs can lead to a very complex model. There are more possible structures to choose from and more parameters are required to obtain a good fit.

2.1. Linear model representations

Based on a linear regression form

\[ y(t) = \phi^T(t) \theta + e(t), \]

where

\[ \phi^T(t) = \begin{pmatrix} \phi^T(t) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \phi^T(t) \end{pmatrix} \]  

(2)
and
\[ \varphi^T(t) = (y^T(t-1) \ldots y^T(t-n_y)u^T(t-1) \ldots u^T(t-n_u)) \] (3)

with
\[ y^T(t-1) = [y_1(t-1)y_2(t-1) \ldots y_{na}(t-1)]^T, \]
\[ y^T(t-n_y) = [y_1(t-n_y)y_2(t-n_y) \ldots y_{na}(t-n_y)]^T, \]
\[ u^T(t-1) = [u_1(t-1)u_2(t-1) \ldots u_{nb}(t-1)]^T, \]
\[ u^T(t-n_u) = [u_1(t-n_u)u_2(t-n_u) \ldots u_{nb}(t-n_u)]^T, \]

where \( y \) and \( u \) are the output and input variables, respectively, \( n_y \) and \( n_u \) are the output and input lags, respectively, \( na \) is the number of outputs and \( nb \) is the number of inputs. The estimated parameters are
\[ \theta = [\theta^1 \theta^2 \ldots \theta^{nu}]^T, \] (4)

where \( \theta^1, \theta^2, \) etc., are the coefficient of the regressors.

Parameter estimation is straightforward if the structure of the model is known. However, there are many processes where the structures describing the equation of the models are not known. For linear models, the determination of the model structure will be the same for SISO system, which consists of choosing the order of the input, output and noise lags. For non-linear models, the possible number of the terms to be included in the model increases as \( n_y, n_u \) and \( n_e \) increase as well as the order of non-linearity \( l \). The problem is compounded for MIMO system where the system is MIMO. For example if \( na = nb = 2 \) and \( n_y = n_u = n_e = l = 2 \), there will be 182 possible terms. Many of the terms may be redundant and including them in the model will create large estimation problem that will lead to a complex model, which is not easy to use and analyse.

2.2. NARX model

Leontaritis and Billings [26] described a deterministic non-linear discrete-time (NARMAX) multivariable in system with \( na \) outputs and \( nb \) inputs as NARX model as
\[ y_i(t) = f_i(y_1(t-1), \ldots, y_{l}(t-n^i_{y}), \ldots, y_{na}(t-1), \ldots, u_{i}(t-n^i_{u}), \ldots, u_{nb}(t-1), \ldots, u_{nb}(t-n^i_{u}) + e_i(t) \]
\[ i = 1, \ldots, na, \] (5)

where \( f_i \) is the unknown non-linear function. The expansion of this NARX model is given as the summation of terms with degree of non-linearity in the range of \( 1 \leq l \leq L \) as follows:
\[ y_i(t) = \sum_{l=1}^{L} y_{i,l}(t), \quad i = 1, 2, \ldots, na \] (6)

with
\[ y(t) = \sum_{l=0}^{L} \sum_{p=0}^{l} \sum_{n_1,n_t} e_{p,l-p}(n_1, \ldots, n_t) \prod_{j=1}^{p} y(t-n_j) \prod_{j=p+1}^{l} u(t-n_j), \] (7)

where each \( l \)th degree of non-linearity term consists of \( p \)th degree factor in \( y(t-n_j) \) and a \((l-p)\)th degree factor in \( u(t-n_j) \), multiplied by \( e_{p,l-p}(n_1, \ldots, n_t) \) and
\[ \sum_{n_1,n_t} \equiv \sum_{n_1=1}^{n_1} \cdots \sum_{n_t=1}^{n_t}, \] (8)

where the upper limit is \( n_y \) and \( n_u \), respectively, if the summation refers to factors in \( y(t-n_j) \) or \( u(t-n_j) \). If the polynomial degree of the \( i \)th sub-system model is \( L_i \), the number of maximum terms is given by the equation
\[ n_i = \sum_{j=0}^{L_i} n_{ij}, \quad n_{0i} = 1, \quad i = 1, \ldots, na \]
and
\[
n_{ij} = n_{ij-1} \left[ \sum_{k=1}^{na} n_{yk}^k + n_{ek}^k + \sum_{k=1}^{nb} n_{uk}^k + j - 1 \right], \quad j = 1, \ldots, L_i. \tag{9}
\]

Model selection may be carried out on the full model and its reduced variants until the optimal model that minimises some statistical criteria is found. An over parameterised model structure can lead to complicated computations for parameter estimation [27]. To determine an adequate approximation of the system that uses as few terms as possible, it might be necessary to test all possible structures in order to find the optimal structure, which sometimes may lead to an exhaustive search. Therefore, a search based on MA is proposed for MIMO non-linear identification and is discussed further in the next sections.

3. Identification of a multivariable system based on MA

The study in this paper is concerned with the generation of non-linear model capable of acting as process simulator for a multivariable system where given the past input and output data, the model should be able to predict the outputs of the process over a long period of time. Given the simplicity and effectiveness of MA in SISO system [28], a similar approach could also prove effective for MIMO systems and is discussed in this section.

3.1. Modified genetic algorithm (MGA) and local search

GA is an optimisation algorithm based on the Darwinian principle of survival of the fittest. The advantage of SGA in this computation is that this algorithm will converge the population into highly fit individuals and these subsets shall be examined in detail to select the fittest among the selected population for the final model structure [29]. The main disadvantage of SGA is premature convergence that leads to poor performance of SGA. An improved strategy for selecting and exploring potential regions in SGA called MGA has been proposed and showed promising results [14,28,29]. In this study, local search technique is added to the proposed MGA and is called MA and has been shown to give better results for identification of SISO systems [28,29].

3.2. Model structure selection algorithm memetic algorithm

Model structure selection using MA has been used as an alternative to determine the model structure of the system. The procedure of the algorithm is shown as a flowchart in Fig. 1. The algorithm attempts to optimise the model structure of multivariable systems and the procedure remains the same as for SISO system as explained in [28] with some additional steps for the MIMO procedure.

3.3. Model validity tests

Model validation is the final procedure in system identification. The objective is to check whether the model fits the data adequately without any bias. Expanding the one-step-ahead prediction (OSA) of SISO systems into its MIMO form gives
\[
\hat{y}_{\text{OSA}}(t) = \hat{F}[[y_i(t-1), \ldots, y_i(t-n_i), u_j(t-1), \ldots, u_j(t-n_j)]],
\]
where \(i = 1, \ldots, na\) and \(j = 1, \ldots, nb\). Similarly, the model predicted output (MPO) is defined as
\[
\hat{y}_{\text{MPO}}(t) = \hat{F}[[\hat{y}_{\text{MPO}}(t-1), \ldots, \hat{y}_{\text{MPO}}(t-n_i), u_j(t-1), \ldots, u_j(t-n_j)]],
\]
where the model output is based on the past predicted output and input data. Unlike OSA which uses observed data at time \(t\) to predict output at time \(t+1\), MPO is an efficient model assessment because the prediction errors at previous time instants are inherited by the predictions at later time instant making it more sensitive to the unmodelled terms and incorrect model structure [29,31]. The correlation tests to validate
MIMO non-linear system are given by these equations [8]:

\[
\begin{align*}
\phi_{x_iy_j}(\tau) &= \delta(\tau), \quad \tau = 0 \quad i = 1, \ldots, na \text{ and } j = 1, \ldots, na, \\
\phi_{u_iy_j}(\tau) &= 0 \quad \text{for all } \tau \quad i = 1, \ldots, nb \text{ and } j = 1, \ldots, na, \\
\phi_{x_i(u_ju_k)}(\tau) &= 0, \quad \tau \geq 0 \quad i = 1, \ldots, na \text{ and } j = 1, \ldots, na \\
&\quad \text{ and } k = 1, \ldots, nb, \\
\phi_{(u_iu_j)v_k}(\tau) &= 0 \quad \text{for all } \tau \quad i = 1, \ldots, nb \text{ and } j = 1, \ldots, nb \\
&\quad \text{ and } k = 1, \ldots, na, \\
\phi_{(u_iu_j)v_kl}(\tau) &= 0 \quad \text{for all } \tau \quad i = 1, \ldots, nb \text{ and } j = 1, \ldots, nb \\
&\quad \text{ and } k = 1, \ldots, na \text{ and } l = 1, \ldots, na.
\end{align*}
\]

(12)
Another performance measure is called Error Index (EI) given by

\[
\text{Error Index} = \sqrt{\frac{\sum (y(t) - \hat{y}(t))^2}{\sum y^2(t)}},
\]

where the performance of the model is also evaluated using normalised root mean square of the residual. The value ranges between 0 and 1 and it gives the measure of relative closeness between the predicted output and the measured output.

### 3.4. Validation of the algorithm

The effectiveness of the proposed algorithm is investigated for various simulated MIMO systems before implementing it to an experimental data. Two discrete-time multivariable non-linear systems were simulated to produce input–output data to be used for structure selection using the algorithm as follows:

**System 1:**
\[
y_1(t) = 0.5y_1(t-1) + u(t-2) + 0.1y_2(t-1)u(t-1) + e(t),
\]
\[
y_2(t) = 0.9y_1(t-1) + u(t-1) + 0.2y_3(t-1)u(t-2) + e(t),
\]

**System 2:**
\[
y_1(t) = 0.5y_1(t-2) + u_1(t-2) + 0.1y_2(t-1)u_1(t-1) + e(t),
\]
\[
y_2(t) = 0.9y_2(t-2) + u_2(t-1) + 0.2y_3(t-1)u_2(t-2) + e(t),
\]

where \(e(t)\) is a white noise in the range of \([-0.01, 0.01]\). The input–output data consist of 500 measurements. These known structure models are selected to validate the ability for the algorithm to determine the correct model structure. Non-linear models with the values for \(n_y, n_u, l\) equal to 2 were used to fit these data. System 1 is a one-input and two-output system and the maximum number of terms is 14 that give 16,383 possible model structures. System 2 is a two-input two-output system and the maximum number of terms is 27 with 134,217,727 possible model structures. The parameters used in the algorithms are 5, 0.6 and 0.01 for population size, \(p_c\) and \(p_m\), respectively.

After 30 generations, the algorithm yields the model given in Tables 1 and 2 with the values of error index for each output. The results show that the algorithm is able to identify the correct model structures for both systems. The simulation results indicate that the algorithm which was initially applied to SISO systems is capable to identify the model structures of these multivariable systems.

A number of case studies have been conducted to study the applications of the proposed algorithm on experimental data of real systems [30]. However, only one case study is reported here. The goal of the simulation study is to investigate the effectiveness of the algorithm in modelling MIMO systems based on the experimental data where their exact structures are unknown.

### 4. Modelling continuous stirred tank reactor (CSTR)

In a chemical process, a model can be developed based on mass and energy balance equations, reaction rates, transport rates for heat and other chemical or physical relationships [25]. This type of modelling is time consuming and expensive and may result with a complex model which sometimes is not easily interpretable by
designers to establish a control policy. An alternative modelling technique is to look initially at the system in total, record the response of the process after applying a known input and use the collected data for determination of data-based description of the system or based on system identification procedure. The formulated mathematical description of the system can therefore provide basis for analysis, design and prediction while the essential causal of input and output relationship can still be captured [2]. Furthermore, model development and simulation time is much reduced compared with developing theoretical or physical models.

One of the most important unit operations in a chemical process is a chemical reactor, which is a non-linear system and where the chemical reaction in the reactor is either exothermic or endothermic. An example of a chemical reactor is a perfectly mixed, jacketed continuously stirred tank reactor or CSTR. The CSTR process is a multivariable system, with MIMO control configuration [7]. In this study, a perfectly mixed, continuously stirred tank reactor with exothermic reactions is considered and is referred as System 3. The theoretical details of this process can be found in [32].

System identification is a technique of modelling the relation between the input and output data without the knowledge of the above equations and it is much less expensive to develop. The ability to develop reliable descriptive models using input–output model at low cost is very useful in process monitoring and prediction. In this section, a non-linear identification technique is employed based on MA as described before to model the relation between the coolant jacket temperature and the reaction temperature and reaction concentration.

4.1. Jacketed continuous stirred tank reactor

In a jacketed CSTR, a fluid stream is continuously fed to the reactor and another fluid stream is continuously removed from the reactor. A cooling jacket surrounding the reactor removes the heat generated by the reaction. In this case study, the input and output data from a pilot scale chemical reactor designed by Hussain et al. [33] have been used for the identification purposes. As the reactor is perfectly mixed, the concentration and temperature at the exit stream are the same as the reactor fluid. The heat of the reaction generated by the process is simulated by a controlled and regulated saturated steam flow rate through a steam coil emerged in the reactor. A jacket surrounding the reactor also has feed and exit streams assumed to be perfectly mixed. Energy produced in the reactor process passes through the walls into the jacket and it removes any heat generated in the tank.

The coolant jacket temperature $T_c$ acting as the input is directly manipulated to control the CSTR temperature $T_r$ acting as one of the output. $T_r$ responses instantaneously when different set points of coolant jacket temperature are set. By manipulating the coolant jacket temperature $T_c$, various output data were obtained. A single-input two-output data set was collected from this experiment. The input $u(t)$ is the coolant jacket temperature set point. The outputs are: $y_1(t)$ as the reactor temperature and $y_2(t)$ as the reactant concentration. There are three sets of experimental data used in this experiment, the first set of data consists of 274 data and it is used for the estimation set. The other two sets of data, which consist of 258 and 355 data sets, respectively, are used for the tests set. All the data are normalised data where the values are between 0 and 1. The process parameters are given in Table 3.

<table>
<thead>
<tr>
<th>Output</th>
<th>Terms</th>
<th>Estimates</th>
<th>Error index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$y_1(t-1)$</td>
<td>0.4993</td>
<td>0.0088</td>
</tr>
<tr>
<td></td>
<td>$u_1(t-2)$</td>
<td>0.9995</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_2(t-1)u_1(t-1)$</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$y_2(t-2)$</td>
<td>0.8999</td>
<td>0.0039</td>
</tr>
<tr>
<td></td>
<td>$u_2(t-1)$</td>
<td>0.9999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_2(t-1)u_2(t-2)$</td>
<td>0.1997</td>
<td></td>
</tr>
</tbody>
</table>
4.2. Process model

Most chemical processes are non-linear, and non-linear mathematical models can adequately describe their behaviour by either using principle of material and energy balance or based on process input and output information. For the jacketed CSTR, the models are developed using the material and energy balance inside the reactor where the process can be described by an ordinary differential equation (ODE). The advantages of modelling using the physical models are the ability to describe the internal dynamics of the process and to include mathematical description of the process. However, there are some disadvantages associated with this type of modelling such as the high cost of model development as well as the limitations of including details of the process due to the lack of information about various model parameters.

Table 3
Process parameters for jacketed CSTR [25]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>0.16 m$^3$/h</td>
</tr>
<tr>
<td>$V$</td>
<td>0.16 m$^3$</td>
</tr>
<tr>
<td>$K_0$</td>
<td>4,878,000 h$^{-1}$</td>
</tr>
<tr>
<td>$(\Delta H)$</td>
<td>20,923 kcal/mole</td>
</tr>
<tr>
<td>$UA$</td>
<td>185 kcal/h K</td>
</tr>
<tr>
<td>$C_{uf}$</td>
<td>25 kg mol/m$^3$</td>
</tr>
<tr>
<td>$T_c$</td>
<td>303 K</td>
</tr>
<tr>
<td>$\rho C_p$</td>
<td>1000 kcal/(m$^3$ K)</td>
</tr>
<tr>
<td>$R$</td>
<td>1.987 cal/mol K</td>
</tr>
<tr>
<td>$E/R$</td>
<td>$5.96 \times 10^3$ K</td>
</tr>
<tr>
<td>$T_0$</td>
<td>318.15 K</td>
</tr>
<tr>
<td>$C_u$</td>
<td>24.51 kg mol/m$^3$</td>
</tr>
<tr>
<td>$T$</td>
<td>310 K</td>
</tr>
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</table>

Table 4
Identified model for System 3 using MA

<table>
<thead>
<tr>
<th>Output</th>
<th>Terms</th>
<th>Estimates</th>
<th>Error index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2.607</td>
<td>0.0496</td>
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<td></td>
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<td>-1.400</td>
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</tr>
<tr>
<td></td>
<td>$y_1(t-1)y_2(t-2)$</td>
<td>-3.255</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_1(t-1)u(t-1)$</td>
<td>0.367</td>
<td></td>
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<tr>
<td></td>
<td>$y_1(t-1)u(t-2)$</td>
<td>-0.214</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_2(t-2)$</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>$y_2(t-2)u(t-2)$</td>
<td>0.153</td>
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<td></td>
<td>$y_3(t-1)$</td>
<td>-3.253</td>
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<td>$y_3(t-1)u(t-1)$</td>
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<td></td>
<td>$y_3(t-1)u(t-2)$</td>
<td>0.685</td>
<td></td>
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<td></td>
<td>$y_3(t-2)u(t-1)$</td>
<td>1.282</td>
<td></td>
</tr>
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<td>$y_3(t-2)u(t-2)$</td>
<td>-0.704</td>
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<td>$u_2(t-1)$</td>
<td>-0.147</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$y_1(t-1)$, $y_2(t-1)$</td>
<td>1.145</td>
<td>0.0464</td>
</tr>
<tr>
<td></td>
<td>$y_3(t-1)$</td>
<td>-0.937</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_1(t-1)u(t-1)$</td>
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<tr>
<td></td>
<td>$y_1(t-1)u(t-2)$</td>
<td>-0.182</td>
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<td>$y_3(t-1)$</td>
<td>0.993</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_2(t-1)u(t-1)$</td>
<td>-0.100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_2(t-2)u(t-2)$</td>
<td>-0.094</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$u_2(t-2)$</td>
<td>0.168</td>
<td></td>
</tr>
</tbody>
</table>
4.3. Identification of CSTR using MA

The model was identified as non-linear models with $l_i = 2, n'_i = n''_i = 2$, where $i = 1, 2$, $na = 2$ and $nb = 1$. The maximum number of model terms is 27 and for each output there are 134,217,727 possible models to be selected from. The GA parameters used in the algorithms are 5, 0.6 and 0.01 for population size, $p_c$ and $p_m$, respectively.

The identified model structures for sub-system 1 and sub-system 2 are given in Table 4 with $EI$ values for both sub-systems equal to 0.0496 and 0.0464, respectively. A comparison between the OSAs and the actual outputs as shown in Figs. 2 and 3 show that they are almost identical and the prediction errors between the actual and the predicted outputs are small. These results show that the predicted outputs are adequate since the model outputs agree with the system outputs as shown in these figures.

![Fig. 2. Reactor temperature of the system $y_1$ superimposed on predicted reactor temperature $\hat{y}_1$ and its residual plot.](image)

![Fig. 3. Reactant concentration of the system $y_2$ superimposed on predicted reactant concentration $\hat{y}_2$ and its residual plot.](image)
4.4. Validation tests

Once the models have been identified and the parameters have been estimated, it is important to know whether the models have successfully captured the system dynamics. Therefore, it is important to evaluate the adequacy and validity of the identified models. Finally, some validation tests are conducted on the identified models that are the MPOs as shown in Figs. 4 and 5, correlation tests as shown in Fig. 6 and cross-validation tests and results are shown in Figs. 7–10.

![Fig. 4. Model predicted output and its residual plot for outputs $y_1$ of System 3.](image1.png)

![Fig. 5. Model predicted output and its residual plot for outputs $y_2$ of System 3.](image2.png)
Fig. 6. Correlation tests for System 3.
The MPO tests as shown in Figs. 4 and 5, respectively, are good because the MPOs follow the system outputs very well. The values for error index for the models are 0.0770 and 0.0583 for outputs 1 and 2, respectively, which are relatively small. The correlation tests as shown Fig. 6 confirm that the models are adequate because the dash lines for the upper and lower bounds that indicate a deviation of 5% show that the tests lie within the 95% confidence band. To further validate the models, two different sets of collected data are used for tests sets or cross-validation tests and the results are illustrated in Figs. 7–10. In order to give a quantitative measurement, the values of $EI$ for those models are calculated. The values are 0.0449 for model output 1, 0.0435 for model output 2 using the second data set. The values of error index are 0.0428 for model output 1 and 0.0277 for model output 2 using the third data set. These validation results also confirm that the algorithm provide good approximations for modelling the jacketed CSTR.
5. Conclusion

The results in this study show that the proposed algorithm provides an efficient way of determining the model structure of unknown multivariable non-linear systems. The results revealed that in each case the algorithm is capable of identifying an adequate model of these multivariable systems based solely on the observed input and output data indicating that the algorithm can be used as an alternative for effective modelling of unknown multivariable linear and non-linear systems. Furthermore, the validation results show that the algorithm is able to identify adequate model that gives excellent predictive output performance where for each case, the OSA of the models was plotted and compared with the actual output. Besides, correlation
tests results for the turbo-alternator and the jacketed CSTR reveal that the fitted models are almost unbiased and have correctly captured the system dynamic. The results presented show that the algorithm can be used as an alternative for predicting the dynamic behaviour of some systems or processes.

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