THREE DAY SHORT COURSE

Course Presentation

An introduction on Offshore Engineering and Technology

INSTRUCTOR: DR. SUBRATA K. CHAKRABARTI

University of Illinois at Chicago
Offshore Structure Analysis Inc.
Plainfield, ILLINOIS
Overview of the 3-Day Course

COURSE OUTLINE

--------- DAY 1 ---------

Morning

Introduction to Design
- History and current state of the art
- Design cycle of offshore structures
- Preliminary concept design

Afternoon

Environment & Construction:
- Offshore environment
- Construction and launching

--------- DAY 2 ---------

Morning

Design Tools and Evaluation
- Static and dynamic stability
- Environmental forces

Afternoon

Design Tools and Evaluation
- Structure response and sea-keeping

--------- DAY 3 ---------

Morning

Design Tools and Evaluation
- Probabilistic design
- Extreme load & strength & fatigue
- Anchoring and mooring system

Afternoon

Design validation and model testing
- Riser system
- Scaling laws & Model testing
- Challenges in deepwater testing
- Assessment of validation methods
History and current state of the art of offshore structures

Definition of Offshore Structures

No fixed access to dry land
May be required to stay in position in all weather conditions

Functions of Offshore Structures

♦ Exploratory Drilling Structures:
  A Mobile Offshore Drilling Unit [MODU] configuration is largely
determined by the variable deck payload and transit speed requirements.

♦ Production Structures:
  A production unit can have several functions, e.g. processing, drilling,
  workover, accommodation, oil storage and riser support.

♦ Storage Structures:
  Stores the crude oil temporarily at the offshore site before its transportation
to the shore for processing.
Types of Offshore structures

Fulmar jacket platform

Jack-up Drilling Unit with Spud Cans
Gravity Base Structures placed on the seafloor and held in place by their weight

Troll A gas platform, world's tallest concrete structure in North Sea
Structures piled to the seabed

500,000 barrel oil storage gravity & piled structure in Persian Gulf

Compliant Tower

- Deck
- Pile top weld connected to the tower at top
- Tower Frame
- Ungrouted pile sliding inside the tower leg or pile guides
Floating Offshore Structures moored to the seabed

Articulated Towers Ready for Tow in the 1970s

Floating structure types

Tension Leg Platform: vertically (taut) moored (by tendons) compliant platform
Semisubmersible Platform: Semisubmersibles are multi-legged floating structures

Marine 7000 4th generation semi-submersible

Floating Production Storage and Offloading (FPSO) ship shaped with a turret

Balder: first "permanent" FPSO in the North Sea

Internal bow turret  Cantilevered bow turret
Deepwater floater types

Ultra deepwater (>1500m) wells drilled in the Gulf of Mexico

![Chart showing the number of wells spudded by year for different types of floating platforms: Compliant, TLP, Semi, SPAR, FPSO. The chart shows a significant increase in the number of wells spudded from 1992 to 2001, with a peak in 2001.](image)

- **Compliant**
- **TLP**
- **Semi**
- **SPAR**
- **FPSO**

Tension leg platforms installed as of 2002

![Diagram showing a cross-section of the Gulf of Mexico with various tension leg platforms.](image)

*Notes:*
1. Year indicates year of first production.
Progression of Spars

5th generation drilling semi-submersible “Deepwater Nautilus”

SeaStar MiniTLP for Typhoon Field
Failure of Offshore Structures

Global failure modes of concern are

- capsizing/sinking
- structural failure
- positioning system failure

Examples of accidents which resulted in a total loss.

Alexander Kielland platform -- fatigue failure of one brace, overload in 5 others, loss of column, flooding into deck, and capsizing

Ocean Ranger accident sequence -- flooding through broken window in ballast control room, closed electrical circuit, disabled ballast pumps, erroneous ballast operation, flooding through chain lockers and capsizing.

Piper Alpha suffered total loss after: a sequence of accidental release of hydrocarbons, as well as escalating explosion and fire events.
Structural damage due to environmental loads

P-36 sank after 6 days after: an accidental release of explosive gas, burst of emergency tank, accidental explosion in a column, progressive flooding, capsizing and.

Gulf Oil Platform -- ThunderHorse
Saturday, July 30, 2005

The BP Thunderhorse semi took a hit from Hurricane Dennis listing 20-30 degrees with the topsides almost in the water. The platform costs over a billion dollars to build.
Design cycle of offshore structures

Design Spiral (API RP2T)
Preliminary concept design

Factors Affecting Field Development Concepts and Components
Offshore environment
Construction process, launching and operations

Construction of Caisson
Construction of the Spar (Technip Offshore)

Seastar® fabrication, Louisiana (SBM Atlantia)

Offloading of Spar Hull for Transport (Dockwise)
Truss Spar in Dry Tow

Holstein Spar being towed out of site

Brutus TLP
Spar Upending Sequence (Kocaman et al, 1997)
Spar Mooring Line Hookup (Kocaman et al, 1997)

Derrick Barge Setting Deck on Spar Platform (Technip Offshore)
Holstein Spar in place being outfitted

ThunderHorse being launched

ThunderHorse being Towed to place
Installation of the lower turret on FPSO Balder

APL Buoy Turret System
Rendition of the new bridge left of the existing bridge

Final Drafts of Caisson Showing the Mooring Lines Attached
Current Flow past New Caisson behind the Roughened Pier

Location of the Immersed Tunnel of the Busan-Geoje Fixed Link
Immersion of TE into trench

Physical Testing of Rigs Moored above the Trench
OCEAN ENVIRONMENT

- Standing Wave vs. Progressive Wave
- Linear Theory (also called Stokes First Order, Airy)
- Stokes Second Order Theory
- Stokes Fifth Order Theory
- Stream Function Theory
- Measured Wave Properties in Wave Tank
- Offshore Applications of Wave Theory
- Current -- Uniform and Shear
- Wave-current Interaction
- Wind and Wind Spectrum

\(^1\text{NOTE: Chapter 3, Hydrodynamics of Offshore Structures}\)
Idealization of Waves

Actual Ocean Wave

Simple Idealizations to Regular Waves

Sine Wave

Nonlinear Regular Wave
STANDING WAVE VS PROGRESSIVE WAVE

Standing wave: Oscillation of waves generated in confined boundaries with no forward speed. Variation with space and time may be separated in a mathematical description. Examples include wave tank, liquid storage tanks, harbors, lakes etc.

Space and time are uncoupled: \( \sin kx \sin \omega t \) – some points do not see any waves

![Diagram of standing wave at times t₁ and t₁ + T/2](image)

Progressive wave: Waves have a forward speed in addition to oscillation. Variation with space and time is combined in a mathematical description. Waves generated in the open oceans are progressive waves. We discuss progressive waves here.

Space and time are dependent: \( \sin (kx - \omega t) \) – all points see maximum waves

![Diagram of progressive wave at two times t₁ and t₂](image)
WAVE THEORIES

Three Independent Parameters describe all waves so that they must be specified for a wave. They are:

- water depth, \( d \)
- wave height, \( H \)
- wave period, \( T \)

\( d \): uniform water depth based on flat bottom

\( T \): fixed interval of time at which a succession of wave crests passes a given point

\( L \): wave length i.e., length between two consecutive crests

\( c \): celerity or velocity of wave propagation

\[ c = \frac{L}{T} \]
WAVE THEORY DEVELOPMENT

The theory is described in terms of

A Boundary Value Problem -- BVP consisting of

A differential equation

certain boundary conditions

it will be shown that

A complete solution of BVP is not possible without approximation.

Approximate solution is obtained through simplified assumptions

Two general types of approximate theory:

(1) wave height as perturbation parameter (deep water)
(2) water depth as perturbation parameter (shallow water)

The theory is formulated in terms of

either

Potential Function, $\Phi$

or

Stream Function, $\Psi$
Basic Assumptions

- Fluid is incompressible

Fluid density = $\rho$.
Fluid is incompressible if $\rho$ is constant. Then the variation of density with time, $t$ is zero:

$$\frac{\partial \rho}{\partial t} = 0$$

since we are dealing with water it is considered incompressible.

- Flow is continuous
- Flow is irrotational

Starting points for the BVP are the following

- Equation of Continuity
- Bernoulli's Equation

Definitions of following important variables to follow:

- Stream Function
- Potential Function
TWO CLASSES OF WAVE THEORIES

A. Closed Form

Linear wave theory
Stokes higher-order wave theory

B. Numerical Solution

Stream Function theory -- Regular
-- Irregular

Properties sought: Fluid particle velocity/acceleration components
Dynamic pressure of fluid particle

\[
\begin{align*}
\mathbf{u}, \mathbf{v}, \mathbf{w} &= \text{velocity components} \\
\mathbf{i}, \mathbf{j}, \mathbf{k} &= \text{unit vectors along } x, y, z
\end{align*}
\]

Equation of Continuity

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

If the above equation is satisfied, then the Continuity of flow is maintained
Stream Function

existence of $\Psi \iff$ continuity

$\Psi$ = constant defines a streamline
Potential Function -- Irrotational Flow

The concept of irrotationality is somewhat abstract/mathematical, but is one of the most important in hydrodynamics.

Flow is called rotational, if each fluid particle undergoes a rotation, in addition to translation and deformation.

\[
\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0
\]

This is the governing differential equation known as LaPlace’s equation.

Similarly, irrotationality \[\nabla^2 \Psi = 0\]
The definition of potential function yields the wave particle kinematics.

We still need a description for the wave particle dynamic pressure. It comes from the Bernoulli's Equation which is derived from the Navier Stokes equation under simplified assumptions.

Assumptions are made of

- Irrotational flow, and
- Perfect fluid – no change in viscosity

Then the wave particle pressure has the expression

\[ p = -\rho g y - \rho \frac{\partial \Phi}{\partial t} - \frac{1}{2} \rho \left[ \left( \frac{\partial \Phi}{\partial x} \right)^2 + \left( \frac{\partial \Phi}{\partial y} \right)^2 + \left( \frac{\partial \Phi}{\partial z} \right)^2 \right] \]

hydrostatic  linear dynamic  nonlinear dynamic
Example:
Consider \( \Phi(y) = A \exp(ky) \) with amplitude \( A = 10 \text{ ft} \), and wave number \( k = 0.01 \). Compute the three pressure components for this wave.
Two-dimensional wave motion

Note that the wave theory is developed over a flat bottom

\( y \) is measured up from the SWL

thus \( y \) is negative below the SWL to the ocean floor

\( s \) is measured up from the floor so that

\[ s = y + d \]
Boundary Value Problem

In terms of velocity potential \( \Phi (x,y,z,t) \), the following equations are obtained in the presence of a body in fluid:

Differential Equation

\[
\nabla^2 \Phi = 0
\]

-- Laplace's equation

Bottom boundary condition

\[
\frac{\partial \Phi}{\partial y} = 0 \quad \text{at } y = -d
\]

Fixed body surface condition

\[
\frac{\partial \Phi}{\partial n} = 0 \quad \text{on the fixed surface}
\]

Moving body surface condition

\[
\frac{\partial \Phi}{\partial n} = V_n \quad \text{on the moving surface}
\]
Linear Wave Theory Development

The following assumption is made:

Waves are two-dimensional

Problem is posed as determining:

either, the velocity potential
or, the stream function

Perturbation technique

Express $\Phi$ as a power series in wave slope (i.e., wave height/ wave length)

$$\varepsilon = \frac{kH}{2} = \frac{\pi H}{L}$$

Perturbation parameter

Then, the velocity potential is expressed as a series

$$\Phi = \sum_{n=1}^{\infty} \varepsilon^n \Phi_n \quad \Phi_n \text{ is the } n^{\text{th}} \text{ order potential}$$

similarly, the free surface profile

$$\eta = \sum_{n=1}^{\infty} \varepsilon^n \eta_n \quad \eta_n \text{ is the } n^{\text{th}} \text{ order profile}$$
Linear Wave Theory (solution is obtained in a closed form)

Dispersion relationship (relates the wave length or wave number to the wave frequency, wave period or wave speed)

\[ c^2 = \frac{g}{k} \tanh kd \quad \text{or} \quad \omega^2 = gk \tanh kd \quad \text{or} \quad L = \frac{g}{2\pi} T^2 \tanh kd \]

Horizontal particle velocity

\[ u = \frac{gkH}{2\omega} \frac{\cosh ks}{\cosh kd} \cos[k(x - ct)] \]

Vertical particle velocity

\[ v = \frac{gkH}{2\omega} \frac{\sinh ks}{\cosh kd} \sin[k(x - ct)] \]

Horizontal particle acceleration

\[ \ddot{u} = \frac{gkH}{2} \frac{\cosh ks}{\cosh kd} \sin[k(x - ct)] \]

Vertical particle acceleration

\[ \dot{v} = -\frac{gkH}{2} \frac{\sinh ks}{\cosh kd} \cos[k(x - ct)] \]

Dynamic pressure

\[ p = \rho g \frac{H}{2} \frac{\cosh ks}{\cosh kd} \cos[k(x - ct)] \]
Deepwater Criterion and Wave Length

Certain simplifications in the mathematical expressions describing properties of linear waves may be made if the water depth is very deep or very shallow.

The following table describes these criteria:

<table>
<thead>
<tr>
<th>Depth of Water</th>
<th>Criterion</th>
<th>$\tanh(kd)$</th>
<th>Deep water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep water</td>
<td>$d/L \geq 1/2$</td>
<td>1</td>
<td>$L_0 = \frac{gT^2}{2\pi}$</td>
</tr>
<tr>
<td>Shallow water</td>
<td>$d/L \leq 1/20$</td>
<td>$kd$</td>
<td>$L = T\sqrt{gd}$</td>
</tr>
<tr>
<td>Intermediate water depth</td>
<td>$1/20 &lt; d/L &lt; 1/2$</td>
<td></td>
<td>$L = \left[ L_0 \tanh(2\pi d / L_0) \right]^{1/2}$</td>
</tr>
</tbody>
</table>

For example, in deep water

Horizontal particle velocity becomes

$$u = \frac{\pi H}{T} \exp(ky) \cos[k(x - ct)]$$

Note the exponential decay term with respect to the vertical coordinate $y$.

Similar expressions follow for the other quantities.
Wave Particle Orbits and Velocity Amplitude Profiles

(a) DEEPWATER \( \frac{d}{L} > \frac{1}{2} \)

(b) INTERMEDIATE DEPTH \( \frac{1}{20} < \frac{d}{L} < \frac{1}{2} \)

(c) SHALLOW WATER \( \frac{d}{L} \leq \frac{1}{20} \)
Sample Calculation of Linear Wave Properties

Water depth, \( d = 100 \text{ ft} \)
Wave Height, \( H = 20 \text{ ft} \)
Wave Period = 10 sec
Elevation, \( y = 0 \text{ ft} \)

Deep water wave length

\[
L_0 = \frac{g T^2}{2\pi} = \frac{5.1248 T^2}{2\pi} = 512.48 \text{ ft}
\]

\(k_0 = 0.01226\)

\[
d/L_0 = 0.1951
\]

\(d/L = 0.2210\)

\(k = 0.01389\)

\[
sinh \; kd = 1.88; \quad cosh \; kd = 2.13
\]

\(u_m = \pi \cdot \frac{20}{10} \times \frac{2.13}{1.88} = 7.12 \text{ ft/sec}\)

\(v_m = \pi \cdot \frac{20}{10} = 6.28 \text{ ft/sec}\)

\(\dot{u}_m = 2\pi \cdot 10 \times 7.12 = 4.47 \text{ ft}^2/\text{sec}\)

\(\dot{v}_m = 2\pi \cdot 10 \times 6.28 = 7.12 \text{ ft}^2/\text{sec}\)
Time History of Linear Wave Properties

Note the phase relationships among variables

\[ H = 20 \text{ ft.} \quad d = 100 \text{ ft.} \]
\[ T = 10 \text{ sec.} \quad y = 0 \]
Exercise 1

Consider a regular wave of 20 ft and 10 sec. in 400-ft water depth. Using linear theory compute the amplitude of velocity and acceleration profiles with depth.

Calculations are carried out on an excel spreadsheet

<table>
<thead>
<tr>
<th>constants</th>
<th>unit</th>
<th>finite depth</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\pi$ = 6.2832</td>
<td>ft$^2$-s</td>
<td>d = 200</td>
<td>ft</td>
</tr>
<tr>
<td>g = 32.2</td>
<td>slug/ft$^3$</td>
<td>H = 20.0</td>
<td>ft</td>
</tr>
<tr>
<td>$\rho$ = 1.98</td>
<td>s</td>
<td>T = 10</td>
<td>s</td>
</tr>
<tr>
<td>$dy$ = -10</td>
<td>ft</td>
<td>$\omega$ = 0.6283</td>
<td>1/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>ft/s</th>
<th>ft/s</th>
<th>ft/s$^2$</th>
<th>ft/s$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>6.38</td>
<td>6.29</td>
<td>4.01</td>
<td>3.95</td>
</tr>
<tr>
<td>-10.0</td>
<td>5.64</td>
<td>5.54</td>
<td>3.54</td>
<td>3.48</td>
</tr>
<tr>
<td>-20.0</td>
<td>4.99</td>
<td>4.88</td>
<td>3.14</td>
<td>3.07</td>
</tr>
<tr>
<td>-30.0</td>
<td>4.42</td>
<td>4.30</td>
<td>2.78</td>
<td>2.70</td>
</tr>
<tr>
<td>-40.0</td>
<td>3.92</td>
<td>3.78</td>
<td>2.46</td>
<td>2.37</td>
</tr>
<tr>
<td>-50.0</td>
<td>3.48</td>
<td>3.32</td>
<td>2.19</td>
<td>2.08</td>
</tr>
<tr>
<td>-60.0</td>
<td>3.09</td>
<td>2.91</td>
<td>1.94</td>
<td>1.83</td>
</tr>
<tr>
<td>-70.0</td>
<td>2.75</td>
<td>2.55</td>
<td>1.73</td>
<td>1.60</td>
</tr>
<tr>
<td>-80.0</td>
<td>2.46</td>
<td>2.22</td>
<td>1.54</td>
<td>1.40</td>
</tr>
<tr>
<td>-90.0</td>
<td>2.20</td>
<td>1.93</td>
<td>1.38</td>
<td>1.21</td>
</tr>
<tr>
<td>-100.0</td>
<td>1.98</td>
<td>1.67</td>
<td>1.24</td>
<td>1.05</td>
</tr>
<tr>
<td>-110.0</td>
<td>1.78</td>
<td>1.44</td>
<td>1.12</td>
<td>0.90</td>
</tr>
<tr>
<td>-120.0</td>
<td>1.62</td>
<td>1.23</td>
<td>1.02</td>
<td>0.77</td>
</tr>
<tr>
<td>-130.0</td>
<td>1.48</td>
<td>1.04</td>
<td>0.93</td>
<td>0.65</td>
</tr>
<tr>
<td>-140.0</td>
<td>1.36</td>
<td>0.86</td>
<td>0.85</td>
<td>0.54</td>
</tr>
<tr>
<td>-150.0</td>
<td>1.26</td>
<td>0.70</td>
<td>0.79</td>
<td>0.44</td>
</tr>
<tr>
<td>-160.0</td>
<td>1.18</td>
<td>0.55</td>
<td>0.74</td>
<td>0.34</td>
</tr>
<tr>
<td>-170.0</td>
<td>1.13</td>
<td>0.40</td>
<td>0.71</td>
<td>0.25</td>
</tr>
<tr>
<td>-180.0</td>
<td>1.08</td>
<td>0.26</td>
<td>0.68</td>
<td>0.17</td>
</tr>
<tr>
<td>-190.0</td>
<td>1.06</td>
<td>0.13</td>
<td>0.67</td>
<td>0.08</td>
</tr>
<tr>
<td>-200.0</td>
<td>1.05</td>
<td>0.00</td>
<td>0.66</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Kinematic amplitudes with depth

d = 200
H = 20
T = 10

-200
-180
-160
-140
-120
-100
-80
-60
-40
-20
0

0.0 2.0 4.0 6.0 8.0

u, v

0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-200

0.0 1.0 2.0 3.0 4.0

u, v

u dot
v dot
Non-dimensional horizontal particle velocity

\[
\bar{u}_0 = \frac{u_0 T}{2\pi (H / 2)} = \frac{\cosh kd (s / d)}{\sinh kd}
\]

Non-dimensional vertical particle velocity

\[
\bar{v}_0 = \frac{v_0 T}{2\pi (H / 2)} = \frac{\sinh kd (s / d)}{\sinh kd}
\]

Non-dimensional dynamic pressure

\[
\bar{p}_0 = \frac{p_0}{\rho g (H / 2)} = \frac{\cosh kd (s / d)}{\cosh kd}
\]
Comparison of Linear Wave Pressures with Tank Measurements

Water depth = 10 ft
Wave Periods = 1.25 – 3.0 sec
Stokes Second Order Theory

\[ \Phi = \varepsilon \Phi_1 + \varepsilon^2 \Phi_2 \]

Wave profile

\[ \eta = \frac{H}{2} \cos(kx - \omega t) \]

\[ + \frac{\pi H^2}{8L} \frac{\cosh kd}{\sinh^3 kd} [2 + \cosh 2kd \cos 2(kx - \omega t)] \]

Horizontal particle velocity

\[ u = \frac{\pi H}{T} \frac{\cosh ks}{\sinh kd} \cos(kx - \omega t) \]

\[ + \frac{3}{4c} \left( \frac{\pi H}{T} \right)^2 \frac{\cosh 2ks}{\sinh^4 kd} \cos 2(kx - \omega t) \]

Vertical particle velocity

\[ v = \frac{\pi H}{T} \frac{\sinh ks}{\sinh kd} \sin(kx - \omega t) \]

\[ + \frac{3}{4c} \left( \frac{\pi H}{T} \right)^2 \frac{\sinh 2ks}{\sinh^4 kd} \sin 2(kx - \omega t) \]

Dynamic pressure

\[ p = \rho g \frac{H}{2} \frac{\cosh ks}{\cosh kd} \cos[kx - \omega t)] \]

\[ + \frac{3}{4} \rho g \frac{\pi H^2}{L} \frac{1}{\sinh 2kd} \left[ \frac{\cosh 2ks}{\sinh^2 kd} - \frac{1}{3} \right] \cos 2(kx - \omega t) \]

\[ - \frac{1}{4} \rho g \frac{\pi H^2}{L} \frac{1}{\sinh kd} [\cosh 2ks - 1] \]
First and second order components

Wave profile over one cycle

NOTE:
The higher order components yield the vertical asymmetry in the wave profile.
The second order component is less than 4 % of the first order velocity in this example.
Stokes Fifth Order Theory

\[ \Phi = \varepsilon \Phi_1 + \varepsilon^2 \Phi_2 + \varepsilon^3 \Phi_3 + \varepsilon^4 \Phi_4 + \varepsilon^5 \Phi_5 \]

\[ u = \sum_{n=1}^{5} u_n \cosh nk s \cos n(kx - \omega t) \]

first term: \( \cosh ks \cos(kx - \omega t) \)

second term: \( \cosh 2ks \cos 2(kx - \omega t) \)

third term: \( \cosh 3ks \cos 3(kx - \omega t) \)

fourth term: \( \cosh 4ks \cos 4(kx - \omega t) \)

fifth term: \( \cosh 5ks \cos 5(kx - \omega t) \)

Note the presence of five frequencies in the profile.
Example

Given \( d = 100 \) ft. \( H = 50 \) ft. \( T = 10 \) sec.

Calculated \( L = 502.219 \) ft.

<table>
<thead>
<tr>
<th>( \theta = 0 )</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = 32.21 )</td>
<td>27.21</td>
<td>16.36</td>
<td>5.44</td>
<td>-2.36</td>
<td>-8.36</td>
<td>-12.65</td>
<td>-15.26</td>
<td>-17.05</td>
<td>-17.79</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>45</td>
<td>90</td>
<td>180</td>
<td>45</td>
<td>90</td>
<td>0</td>
<td>90</td>
<td>180</td>
</tr>
<tr>
<td>s</td>
<td>u</td>
<td>u</td>
<td>u</td>
<td>u</td>
<td>v</td>
<td>v</td>
<td>p</td>
<td>p</td>
<td>p</td>
</tr>
<tr>
<td>132.21</td>
<td>28.73</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>125.00</td>
<td>25.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>113.93</td>
<td>11.87</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>110.00</td>
<td>20.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.00</td>
<td>18.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>94.06</td>
<td>-2.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>82.21</td>
<td>-10.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80.00</td>
<td>14.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60.00</td>
<td>11.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40.00</td>
<td>9.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.00</td>
<td>8.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>8.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note:

The wave crest is at 132.21 ft while the wave trough is at 82.21 ft.

The asymmetry in the horizontal velocity between the crest and the trough is remarkably pronounced.
STREAM FUNCTION THEORY

Two Types:

(1) Symmetric Regular (theoretical)
(2) Irregular (when the measured profile is known)

Coordinate System
moves with wave speed, \( c \) in the direction of the wave propagation
Horizontal velocity \( = u - c \)

If current is present, add the current velocity, \( U \) to the above
Horizontal velocity \( = u - c + U \)

For irregular waves,
The measured wave profile is used as an additional constraint

Solution taken in a series form:

\[
\Psi(x, y) = cy + \sum_{n=1}^{N} X(n) \sinh nks \cos nkx
\]

where \( N \) is the order of Stream Function wave theory
The unknowns \( X(n) \) are solved by using the dynamic free surface boundary condition
Order of Stream Function NORDER

Based on Wave Parameters d, H, T

This chart is useful in determining a suitable order of the stream function theory in a particular wave case.
Nonlinear Regular Stream Function Wave Profile

compared with

Linear Wave Profile

Wave Height = 195.4 ft (59.6m)
Wave Period = 15 sec
Water Depth = 1152 ft (351.2m)

NOTE: in this figure $h$ represents $d$ = water depth
Irregular Stream Function Wave vs. Experimental Pressure Amplitude

\[ d = 10 \text{ ft}; T > 3.00 \text{ sec} \]
Comparison of Computed Velocities with Measurements

Irregular Stream Function Theory

Water depth = 2.5 ft
H/d = 0.7
Region of application of wave theories
PRACTICAL APPLICATION OF WAVE THEORIES

Wave Theories

- **Linear Theory**
  - Low seastates (1 yr storm)
  - Swell
  - Large inertia dominated fixed and floating structures
  - Linear radiation damping
  - Fatigue analysis
  - Long term statistics

- **Stokes Second Order Theory**
  - Slow drift oscillation of soft moored structures
  - TLP tendon analysis

- **Stokes Fifth Order or Stream Function Theory**
  - Storm waves
  - Drag dominated structures
  - High wave tank data (irregular stream function)
  - Air gap or run-up
  - Deck waves