Power transmission

Components used to transmit power: gears, belt, clutch and brakes.

Gear

Objective:
Student must be able to force analysis, stress analysis using basic formula (Lewis) and AGMA (bending stress and surface stress)

Type of gear:
Spur gear only

Types of Gear

a) Spur Gear

- Teeth is parallel to axis of rotation
- Can transmit power from one shaft to another parallel shaft

b) Helical gear

- Teeth is inclined to the axis of rotation
- Smoother than spur
- Develop thrust load (helix angle)
- Can transmit power from one shaft to a parallel and non-parallel shaft
c) Bevel gear

- Teeth on conical surfaces
- Transmit power between two intersecting shafts


d) Worm gear

- Transmit power between two intersecting shafts

Terminologies

A pair of gears can be represented as 2 circles:

\[ d_1 = Nm \]
\[ d_2 = Nm \]

where: 
- \( N \): number of teeth
- \( m \): module
- Note: Mating gears must have the same module.
**Module**: is the ratio of diametral pitch and number of teeth $m = \frac{d}{N} [\text{mm}]$

**Face Width (F)**: width of the tooth

**Addendum [a]**: distance between top face of the tooth to pitch circle

**Dedendum [b]**: distance between pitch diameter to bottom of the gear

In the following: we only concentrates on **full depth gear**
When the offset occurs between pitch. → not full-depth tooth, which is called stub.

**Undercut**: resulted from number of tooth is less than the minimum number of tooth suggested. Results: higher stresses at the root of the tooth. (Refer to section 13-7 interference)

**Backlash**: gap between mating tooth. The gap can be used for lubrication.
Contact Ratio: the number of tooth in contact during meshing. Roughly \( \rightarrow \) spur gear (1.4 to 1.8)

Pinion and Gear: pinion is driver and gear is driven

Gear Parameters

Metric Module \( m = d/N \)

Imperial Diametral Pitch \( P = N/d \) (inverse to module)

One pair of gear must have the same module

Pressure Angle: \( 20^\circ, 22.5^\circ, 25^\circ \)

Table 13-1: relationship between addendum (a) and dedendum (b) pp 696

Table 13-2: Available pitch diametral and module.
Gear Train

\[ V_2 = V_3 \]

Known that \( v = \frac{\pi dn}{60} \)
where \( n \): revolution / minit

\[ \pi d_2 n_2 = \pi d_3 n_3 \]
\[ d_2 n_2 = d_3 n_3 \]

Equation 1

For a pair of gear \( m_2 = m_3 \)

\[ \frac{d_2}{N_2} = \frac{d_3}{N_3} \]

Equation 2

From Eq 1 and 2

\[ \frac{d_2}{d_3} = \frac{N_2}{N_3} = \frac{n_3}{n_2} \]

Significance:
✓ d increases N increase
✓ d increases n reduces \( \rightarrow \) to reduce rpm requires small pinion and larger gear and vice versa.
Gear Train (continued)

\[ V_2 = V_3 \text{ and } V_3 = V_4 \]

Therefore

\[ V_2 = V_3 = V_4 \]

\[ \frac{N_2}{N_3} = \frac{n_2}{n_3} \]

From previous formula:

\[ N_2 n_2 = N_3 n_3 \]

Gear 2 and 3

\[ n_3 = \frac{N_2}{N_3} n_2 \quad \ldots \ (1) \]

Gear 3 and 4

\[ n_4 = \frac{N_3}{N_4} n_3 \quad \ldots \ (2) \]

Eq (1) in eq (2)

\[ n_4 = \frac{N_3}{N_4} \frac{N_2}{N_3} n_2 \]

\[ n_L = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} n_F \]

where:

\[ n_L : \text{rotational speed of last gear (output)} \]
\[ n_F : \text{rotational speed of first gear (input)} \]

Train value

\[ e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} \]
Planetary Gear

Gear 2: Sun gear
Gear 3: Arm
Gear 4: Planet Gear
Gear 5: Ring Gear

Assumption Arm Fixed:

Train value \( e = \frac{N_2N_4N_5}{N_4N_3N_6} \)
3 MAGIC FORMULAE FOR FBD ANALYSIS ON GEAR

Torque
\[ T = \left( \frac{d}{2} \right) W_t \ [Nm] \]
\[ W_t: \text{tangential force} \]

Speed
\[ V = \frac{\pi D n}{60} \ [m/s] \]
\[ D: \text{pitch diameter in [m]} \]
\[ n: \text{rotational speed [rpm]} \]

Power
\[ H = W_t V \ [watts] \]
Force Analysis (Free Body Diagram)

To transfer power, T must exist.

When the pinion rotates, tooth from gear against the movement \( \rightarrow \) direction \( W_{132} \) must against the direction of rotation

\[
W_{132} = \frac{H}{V}
\]

\[
T_2 = W_{132} \times \frac{d_2}{2}
\]

Due to pressure angle, \( W_{r32} \) (radial force) is generated

\[
W_{r32} = W_{132} \tan \theta
\]
On Gear 3,

$W_{t_{23}}$ and $W_{r_{23}}$ must in the opposite direction.

To be statically analytical, $T_3$ is against $W_{t_{23}}$

$$T_3 = W_{t_{23}} \cdot d_3/2$$

*Note: $W_{t_{23}}$ can be calculated using $W_{t_{32}} = H/V$, please remember that all the parameters must be based on gear 3.*

Discuss example 13-7
Example

You are responsible to design a gear system for speed reducer. The speed reducer is a two stage reduction which each pinion has 18T (Gear 2 and 4 in Figure 1). One of the constraints is that the maximum allowable reduction is 10 at each stage. Based on this, answer the following questions.

a. Suggest the two possible number of teeth for Gear 3 and 5 if the speed has to be reduced by 24 times. Note: if Gear 3 has X teeth and Gear 5 has Y teeth and vice versa, the answer is considered as one)

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Assume gear 3 and 5 have 72 and 90 teeth respectively and comprises of m = 4 and 20° pressure angle gears. The motor is 8kW at 1000 rpm clockwise.

b. Calculate the rpm of the output shaft.

c. Draw and calculate all the resultant forces on all of the gears.

d. Based on the above calculation, discuss the relationship between torque and gear ratio.

![Figure 1](image-url)
Example

The figure below shows a dual output power transmission system. A 8kW motor with 1000 rpm in clockwise direction is attached to shaft 1 at A. 40% of the power is delivered to shaft 2 using gear 2 and 4 and the remaining 60% of the power is delivered to shaft 3 via gear 3 and 5. All the gear module is 2 mm (m = 2mm) with pressure angle of 20°.

Based on this information, answer the following questions

a) Draw the FBD for every gear and also calculate all the forces and torques if the speed of both output shafts have been reduced by 3.

b) Calculate N₅ if torque on shaft 2 and 3 is equal and N₄ is 72.
Failure Types

Bending: resulted from bending stress.
W act on the tooth
Lewis formula and AGMA

Pitting: resulted from surface stress
Repetition of high contact stresses

Scoring: resulted from insufficiency of lubrication

Bending Stresses

Basic Formula:
\[ \sigma = \frac{W'}{FmY} \]

*take note that there are two formulae in pair in the textbook (imperial and metric)
Assumption Basic Formula
- Cantilever Beam Problem
- $F_i$ is carried by one tooth only.

However, dynamic effects are present when a pair of gear at moderate and high speed. ($K_V'$)

$$\sigma = \frac{K'_V W'}{Fm Y} \quad \ldots \text{eq 14-1 pp738}$$

$Y$: Lewis form factor Table 14-2 pp 718. (interpolation if it is required)

### Dynamic Effect

$$K'_V = \frac{3.05 + V}{3.05} \quad \text{(cast iron, cast profile)}$$

$$K'_V = \frac{6.1 + V}{6.1} \quad \text{(cut or milled profile)}$$
\[
K'_{v} = \frac{3.56 + \sqrt{V}}{3.56}
\]  
(hobbed or shaped profile)

\[
K'_{v} = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}}
\]  
(finishing process on gears:  
shaved or ground profile)
Fatigue Strength

Endurance Limit \( S_e = k_a k_b k_c k_d k_e S'_e \)

\( S'_e = 0.5 S_{ut} \)

For steel
\[
S_u = \begin{cases} 
0.5 H_B & \text{kpsi} \\
3.4 H_B & \text{MPa} 
\end{cases} \quad (2-21)
\]

For cast iron
\[
S_u = \begin{cases} 
0.23 H_B - 12.5 & \text{kpsi} \\
1.58 H_B - 86 & \text{MPa} 
\end{cases} \quad (2-22)
\]

AGMA
Surface Factor $k_a$

- \[ k_a = aS_{ut}^b \]

Machining process \[ k_a = 4.51S_{ut}^{-0.265} \]
Size Factor $k_b$

- \[
k_b = \begin{cases} 
1.24d_e^{-0.107} & \text{for } 2.79 \leq d \leq 51\text{mm} \\
1.51d_e^{-0.157} & \text{for } 51 < d \leq 254\text{mm}
\end{cases}
\]

In the case of gear tooth $d = d_e$

\[
d_e = 0.808\sqrt{Ft}
\]

\[
x = \frac{3Ym}{2} \quad \text{(Eq 14-3)}
\]

\[
t = \sqrt{4xl} \quad \text{(Eq b)}
\]

$I = add + dedd$ (refer to Table 13-1)

Stress Concentration Factor

- $K_f = K_{f1}K_{f2}$

Stress concentration factor due to load ($K_{f1}$)

\[
k_{f1} = 1.66 \text{ (one way bending)} \\
k_{f1} = 1 \text{ (two way bending)}
\]

\[
K_{f1} = \frac{1}{k_{f1}}
\]

Stress concentration 2 ($K_{f2}$)
Refer to Table A-15-6

\[ r = 0.3m \]
\[ d = t \]
\[ D = \text{infinite} \]

Based on \( r/d \) and \( D/d = \text{infinite} \), uses the largest \( D/d \), get the value of \( K_t \)

\[ K_{f2} = 1 + q (K_t - 1) \]

**Definition of one and two way bending**

One way bending: tooth subjected one direction rev.
Two way: tooth subjected to both direction rev (forward and reverse)

**Fatigue failure theories**

**Goodman Line**

\[
\frac{K_f \sigma_m}{S_{ut}} + \frac{K_f \sigma_a}{S_e} = \frac{1}{n}
\]

**Gerber Failure**

\[
\left( \frac{nK_f \sigma_m}{S_{ut}} \right)^2 + \frac{nK_f \sigma_a}{S_e} = 1
\]

Determine the mean and alternating stress

One way bending
\[ \sigma_m = \sigma_a = \frac{\sigma}{2} \]

Two way bending

\[ \sigma_a = \sigma \]
\[ \sigma_m = 0 \]

Where \[ \sigma = \frac{K_v W'}{FmY} \]

**Project**

You have to set \( m, F, N_P, N_G \).

The constraint

- Min no of tooth for pinion: Eq 13-11
- Max no of tooth for gear: Eq 13-12
- Face width \( 2p < F < 5p \)
- Material: Figure 14-2, 14-3
SURFACE DURABILITY

Surface Stresses (compressive –ve)

\[ \sigma_c = -C_p \left[ \frac{K_v W_i}{F \cos \phi} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2} \]

\( \phi \) = pressure angle
\( P \) = pinion
\( G \) = gear

\( C_p \) = elastic coefficient

\[ C_p = \left[ \frac{1}{\pi \left( \frac{1-v_P^2}{E_P} + \frac{1-v_G^2}{E_G} \right)} \right]^{1/2} \]

\( v \) = Poisson Ration (Table A-5)
\( E \) = Modulus of Elasticity (Table A-5)

Radius of curvature of the tooth profile

\[ r_1 = \frac{d_P \sin \phi}{2} \]
\[ r_2 = \frac{d_G \sin \phi}{2} \]
Example

A 19-TOOTH 300 Bhn HOBBED STEEL SPUR GEAR PINION TRANSMITS 15 Kw AT A PINION SPEED OF 360 rev/min TO A 77 TOOTH OF THE SAME MATERIAL GEAR. THE FACE WIDTH IS 75 mm, = 20° AND m = 6mm.

a) USING LEWIS FORMULA CALCULATE THE STRESSES DUE TO BENDING AND THE CONTACT STRESSES?

b) CALCULATE THE F.S OF THE BENDING STRESS AGAINST ITS FATIGUE STRENGTH USING GERBER THEORY?

c) CALCULATE THE F.S OF THE CONTACT STRESSES AGAINST CONTACT STRESS ENDURANCE LIMIT (Sₙ)?
Soln

a) Bending Stresses

Pinion

\[ \sigma = \frac{K_v W^t}{FmY} \]

\[ d = Nm = (19)(6) = 114 \text{ mm} = 0.114 \text{ m} \]

\[ V = \frac{\pi dn}{60} = \frac{\pi(0.114)(360)}{60} = 2.149 \text{ m/s} \]

\[ W_i = \frac{H}{V} = \frac{15,000}{2.149} = 6980 \text{ N} \]

Hobbed \( K'_v = \frac{3.56 + \sqrt{V}}{3.56} = 1.402 \)

\[ Y = 0.314 \text{ (Table 14-2)} \]

\[ \sigma = \frac{K_v W^t}{FmY} = 69.8 \text{ MPa} \]

GEAR

\[ \sigma = \frac{K_v W^t}{FmY} = \frac{(1.412)(6980)}{(75)(6)Y} \]

\[ Y \text{ interpolation (77 tooth)} Y = 0.436 \]

\[ \sigma = \frac{K_v W^t}{FmY} = \frac{(1.412)(6980)}{(75)(6)Y} = 50.23 \text{ MPa} \]
b) **Surface Stresses**

\[
\sigma_C = -C_P \left[ \frac{K_v W_t}{F \cos \phi} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2}
\]

\(K_v = 1.40\)

\(W_t = 6980\) N

\(F = 75\) mm

\(d_p = Nm = (19)(6) = 114\) mm

\(d_g = Nm = (77)(6) = 462\) mm

\(r_1 = \frac{d_p \sin \phi}{2} = 19.5\) mm

\(r_2 = \frac{d_g \sin \phi}{2} = 79\) mm

\[
C_P = \left[ \frac{1}{\pi \left( \frac{1-\nu_p^2}{E_p} + \frac{1-\nu_G^2}{E_G} \right)} \right]^{1/2}
\]

Carbon Steel:

\(V = 0.292\)

\(E = 207\) GPa = 207 \(\times 10^3\) MPa

\(C_P = 189.780\) MPa

\[
\sigma_C = -C_P \left[ \frac{K_v W_t}{F \cos \phi} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2}
\]

\[
= -(189,780) \left[ \frac{(1.402)(6980)}{(75)\cos(20)} \left( \frac{1}{19.5} + \frac{1}{79} \right) \right]^{1/2}
\]

\(= -565.5\) MPa
Safety factor

\[ n = \left( \frac{S_c}{\sigma_c} \right) \]

\[ S_c = 6.89(0.4H_B - 10) \text{ MPa} \]

Sc = 757.9

n = 1.15
Limits

\[ \sigma_{all} = \frac{S_o}{F.S} = \frac{k_a k_b k_c k_d k_e S_o}{F.S} \]

*It is advisable to have F.S > 3*

Contact Stress

\[ \sigma_c = -C_p \left[ \frac{K v W^t}{F \cos \phi \left( \frac{1}{r_1} + \frac{1}{r_2} \right)} \right]^{\frac{1}{2}} \]

... Eq 14-14 pp 732

- Negative: because it is always in compression

Cp .... Eq 14.13

E : Modulus of elasticity \hspace{1cm} \text{Table A-5}

\( \nu \) : Poisson’s Ratio \hspace{1cm} \text{Table A-5}

r1, r2 : Eq 14-12
Example

You are responsible to design a gear system for speed reducer. The speed reducer is a two stage reduction which each pinion has 18T (Gear 2 and 4 in Figure 1). One of the constraints is that the maximum allowable reduction is 10 at each stage. Based on this, answer the following questions.

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Assume gear 3 and 5 have 72 and 90 teeth respectively and comprises of m = 4 and 20° pressure angle gears. The motor is 8kW at 1000 rpm clockwise.

b. Calculate the rpm of the output shaft.

c. Draw and calculate all the resultant forces on all of the gears.

d. Based on the above calculation, discuss the relationship between torque and gear ratio.

Figure 1
a)  

<table>
<thead>
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<th>Comb 1</th>
<th>144 (8 x 18)</th>
<th>54 (3 x 18)</th>
</tr>
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<tbody>
<tr>
<td>Comb 2</td>
<td>108 (6 x 18)</td>
<td>72 (4 x 18)</td>
</tr>
</tbody>
</table>

\[
e = \frac{N_2 N_4}{N_4 N_5}
\]

b)  

\[
\begin{align*}
e &= \frac{18 \times 18}{72 \times 90} \\
&= 0.05
\end{align*}
\]

\[
n_L = e \cdot n_F
\]

\[
= (0.05)(1000)
\]

\[
= 50 \text{ rpm}
\]

c) FBD at Gear 2

Diameter pinion \( d = N m \)

\[
d = (18)(4 \times 10^{-3})
\]

\[
= 0.072 \text{ m}
\]

Calculating the tangential force:

\[
F_t = \frac{60H}{\pi d n}
\]

\[
= \frac{60(8000)}{\pi(0.072)(1000)}
\]

\[
= 2122 \text{ N}
\]

\[
= 2122 \text{ kN}
\]

Direction of \( F_t \): when gear 2 pushes gear 3, gear 3 will resist and therefore the direction is UP

\[
F_r = F_t \tan 20^0 = 0.773 \text{ kN}
\]

\[
T_2 = F_t \frac{d}{2} = (2122)(0.072/2) = 76.4 \text{ Nm}
\]
FBD at Gear 3

Reaction at gear 3 is equal in magnitude and opposite direction to gear 2

Diameter \( d_3 = Nm \)
\[ = (72) (4\times10^{-3}) \text{ m} \]
\[ = 0.288 \text{ m} \]

\( T_3 = \frac{F_r d_3}{2} \)
\[ = 2122 (0.288/2) \]
\[ = 305.6 \text{ Nm} \]

Rotation direction ccw

FBD at Gear 4

\( T_3 \) must be equal to \( T_4 \) with opposite direction. This is because the G3 and G4 is on the same shaft, therefore, the total Torque = 0. Rotation direction ccw

\( T_3 = T_4 = 305.6 \text{ Nm} \)

Gear 4 (ccw) pushes gear 3: direction of \( F_t \) is DOWN.

\( F_t = 2 \frac{T}{d} \)
\( d = 0.072 \text{ m} \)
\( F_t = 8.48 \text{ kN} \)
\( F_r = 3.09 \text{ kN} \)

\[
F_t = \frac{60H}{\pi dn}
\]
\[= \frac{60(8,000)}{\pi(0.072)(250)}\]
\[= 8.48 \text{kN} \]
Gear 5

\[ F_i = 3.09 \text{ kN} \]

\[ F_r = 8.48 \text{ kN} \]

\[ T_5 = 1527.8 \text{ Nm} \]