Design for Static Load

Objective
Strengthen the knowledge: static and strength of material.
Know how to do analysis on component under static load
Find the relationship between these topics
Know how to use Table in appendix A

Notation

Axis: Right Hand Rule
- To maintain the position of X, Y and Z axis
- To show the rotation direction of Moment and Torsion

Force: arrow with single head
Eg: $F_x = 10 \text{ kN}, F_y = 20 \text{ kN}$ and $F_z = 30 \text{ kN}$
$F = (10i + 20j + 30k) \text{ kN}$
Moment and torsion: arrow with two head on direction of rotation.
Eg: $M_x = 10 \text{ Nm}$, $M_y = 20 \text{ Nm}$ and $M_z = 30 \text{ Nm}$

\[ M = (10\mathbf{i} + 20\mathbf{j} + 30\mathbf{k}) \text{ Nm} \]

Are torsion and moment same and why they are named differently?
Static load design

\[
\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0 \\
\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0
\]

<table>
<thead>
<tr>
<th>TABLE 1–1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of connection</strong></td>
</tr>
<tr>
<td><img src="image" alt="Cable" /></td>
</tr>
<tr>
<td><img src="image" alt="Roller" /></td>
</tr>
<tr>
<td><img src="image" alt="Smooth support" /></td>
</tr>
</tbody>
</table>

Discuss the following problem

![Figure: 07-015-016P](image)
Based on T, which one has higher force value (F1 or F2). If pulley diameter is D, find F1 in function of D and T.

Find F1 in function of D (sprocket diameter) and d (pulley diameter)
1. 3-80 pp 140

2. 3-84 pp 140
3. 3-68 pp 137

FBD Figure 3-72, 3-73
INDETERMINATE PROBLEM

L1 = 150 mm
L2 = 50 mm
**STRESS ANALYSIS**

**Axial Load (P)**

\[
\sigma = \frac{P}{A}
\]

**Stresses on element**

**Shear load (V)**: can be neglected when the part is subjected to bending or torsion

**Transverse shear stress**

\[
\tau = \frac{V}{A}
\]
Moment

Bending stress \( \sigma = \frac{Mc}{I} = \frac{M}{Z} \)

- \( c \): distance from the center axis
- \( I \): second moment of area (last page, Table A-6 to A-8 pp1008 - 1012)
- \( Z \): section modulus (Table A-6 to A-8)

Stresses on element

\( \sigma = \frac{-My}{I} \)

\( \sigma = \frac{My}{I} \)
In design we are responsible our design is safe against failure. Therefore, parameters such $A$, $I$, $Z$ and $J$ play important role as well as the loads themselves.

Note
We should identify the most critical element in order to proceed with our analysis.

We should calculate the maximum stresses resulted from stress on the element of the most the critical position. How?
Compounded Load

Which element is most critical and why?
<table>
<thead>
<tr>
<th>T</th>
<th>M</th>
<th>Compounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>B, D</td>
<td>B, D</td>
<td>B, D</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>

Which element is most critical and why?
Which element is most critical and why?
**Mohr Circle**
Mohr Circle is used to represent graphically the state of stress on the entire structure.

Element

![Mohr Circle Diagram](image)

Refer to Eq 3-13, 3-14 pp 77, 78 to calculate the principal stress
Read Mohr Circle background theory: Section 3-6 (pp 70 – 84)
STATIC FAILURE THEORIES (Chap 5)

Ductile materials (yield criteria)
- Maximum shear stress (MSS): Sec 5-4 pp 211
- Distortion energy (DE): Sec 5-5 pp 213
- Ductile Coulomb-Mohr (DCM): Sec 5-6 pp 219

Brittle materials
- Maximum normal stress (MNS) Sec 5-8
- Brittle Coulomb-Mohr (BCM) Sec 5-9
- Modified Mohr (MM) Sec 5-9

Maximum shear stress Theory (MSS)

Yielding begins whenever the maximum shear stress in any element equals or exceeds the maximum shear stress in a tension test specimen of the same material when that specimen begins to yield.
Therefore, any Mohr circle generated from the most critical element must lie inside this limit.

Failure occurs when the Mohr circle lies outside the limits.

When the limits are plotted against $s_a$ and $s_b$.

Case 1: when $s_a$ and $s_b$ are $+ve$
Case 2: when $s_a$ is $+ve$ and $s_b$ is $-ve$
Case 3: when $s_a$ and $s_b$ are $-ve$
Distortion Energy Theory (DE)

The effective stress is called Von Misses

$$\sigma' = \sqrt{\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2}$$

$$\sigma' = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$

If $s_y = 0$

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

Therefore failure happens when $s' > S_y$.

To plot the graph $s_a$ against $s_b$ for DE Theory, $s' = S_y$. 

![Graph showing the effective stress based on Von Mises criteria with axes Qa and Qb, and points Sy and Syc indicating stress levels.](image-url)
Load line and its meaning on MSS and DE.
Line when it is drawn from (0, 0) to another point and this line across the \((s_a, s_b)\) of the Mohr Circle of the element.
Intersection between load line and the MSS and DE Curve: point of yielding according the theories defn. Any stresses exceed these intersecting point is considered fail due to yielding \((FS < 1)\).
- Before the intersection point \((FS > 1)\)
- At the intersection point \((FS = 1)\)
- Beyond the intersection point \((FS < 1) \rightarrow NO\ GO\)

Pure Shear Line
When the element is subject to shear only. The intersection of this line with MSS and DE curve will remark the relationship between yield stress and shear yield stress. There
- MSS : \(S_{sy} = 0.5\ Sy\)
- DE : \(S_{sy} = 0.577\ Sy\)
Static failure using mathematical approach

**Three-dimensional Stresses (Section 3-7 pp 82)**

When there is no pressure, $s_z$ is equal to zero, therefore one of principal stress must equal to zero.
What is the value of $s_3$, $s_2$, $s_1$ and $\tau_{\text{max}}$ for a, b and c?

**MSST theory**

Maximum shear

$$\tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2}$$

Allowable shear stress

$$S_{sy} = \frac{S_y}{2n}$$

Safety factor

$$n = \frac{S_y}{\sigma_1 - \sigma_3}$$

DET
Total stress

\[ \sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \]

\[ \sigma' = \sqrt{\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2} \]

Safety factor

\[ n = \frac{S_y}{\sigma'} \]

*NOTE
Please differentiate between

- stresses on element \( \sigma_x, \tau_{xy} \)
- 2D Principal stresses \( \sigma_A, \sigma_B \)
- 3D Principal stresses \( \sigma_1, \sigma_2, \sigma_3 \)

Brittle Material
Figure 5-19


Gray cast-iron data
Note:
Please take note of the following appendix
(All the tables on the appendices should be at your finger tips)

Table A-6 to A-8

Table A-6
Properties of Structural-Steel Equal Legs Angles

<table>
<thead>
<tr>
<th>Size, in</th>
<th>w</th>
<th>A</th>
<th>k₁₋₁</th>
<th>Z₁₋₁</th>
<th>y</th>
<th>k₃₋₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 1 × ⅞</td>
<td>0.80</td>
<td>0.234</td>
<td>0.021</td>
<td>0.298</td>
<td>0.029</td>
<td>0.290</td>
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<tr>
<td>× ⅞</td>
<td>1.49</td>
<td>0.437</td>
<td>0.036</td>
<td>0.287</td>
<td>0.054</td>
<td>0.336</td>
</tr>
<tr>
<td>1 ½ × 1 ½ × ⅞</td>
<td>1.23</td>
<td>0.36</td>
<td>0.074</td>
<td>0.45</td>
<td>0.068</td>
<td>0.41</td>
</tr>
<tr>
<td>× ⅞</td>
<td>2.34</td>
<td>0.69</td>
<td>0.135</td>
<td>0.44</td>
<td>0.130</td>
<td>0.46</td>
</tr>
<tr>
<td>2 × 2 × ⅞</td>
<td>1.65</td>
<td>0.484</td>
<td>0.190</td>
<td>0.626</td>
<td>0.131</td>
<td>0.546</td>
</tr>
<tr>
<td>× ⅞</td>
<td>3.19</td>
<td>0.938</td>
<td>0.348</td>
<td>0.609</td>
<td>0.247</td>
<td>0.592</td>
</tr>
<tr>
<td>× ⅞</td>
<td>4.7</td>
<td>1.36</td>
<td>0.479</td>
<td>0.594</td>
<td>0.351</td>
<td>0.636</td>
</tr>
<tr>
<td>2 ½ × 2 ½ × ⅞</td>
<td>4.1</td>
<td>1.19</td>
<td>0.703</td>
<td>0.769</td>
<td>0.394</td>
<td>0.717</td>
</tr>
<tr>
<td>× ⅞</td>
<td>5.9</td>
<td>1.73</td>
<td>0.984</td>
<td>0.753</td>
<td>0.566</td>
<td>0.762</td>
</tr>
<tr>
<td>3 × 3 × ⅞</td>
<td>4.9</td>
<td>1.44</td>
<td>1.24</td>
<td>0.930</td>
<td>0.577</td>
<td>0.842</td>
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</table>

Table A-9
Shear, Moment, and Deflection of Beams
(Note: Force and moment reactions are positive in the directions shown; equations for shear force V and bending moment M follow the sign conventions given in Sec. 3–2.)

1 Cantilever—end load

\[ R_1 = V = F \quad M_1 = Fl \]
\[ M = F(x - l) \]
\[ y = \frac{Fx^2}{6EI}(x - 3l) \]
\[ y_{max} = \frac{Fl^3}{3EI} \]
Table A-18

### Geometric Properties

#### Part 1: Properties of Sections

- **A**: area
- **I**: location of centroid
- **I_x**: second moment of area about x-axis
- **I_y**: second moment of area about y-axis
- **I_d**: second moment of area about z-axis
- **I_2**: second polar moment of area about z-axis

**Rectangle**

\[
A = bh \\
I_x = \frac{bh^3}{12} \\
I_y = \frac{b^3h}{12} \\
I_d = 0
\]

**Circle**

\[
A = \pi r^2 \\
I_x = I_y = \frac{\pi r^4}{4} \\
I_d = 0 \\
I_2 = \frac{\pi r^4}{2}
\]

**Hollow circle**

\[
A = \pi (r_o^2 - r_i^2) \\
I_x = I_y = \frac{\pi (r_o^4 - r_i^4)}{4} \\
I_2 = 0 \\
I_d = \frac{\pi (r_o^4 - r_i^4)}{2}
\]

**Right triangle**

\[
A = \frac{bh}{2} \\
I_x = \frac{b^3h}{12} \\
I_y = \frac{b^3h}{12} \\
I_d = 0 \\
I_2 = \frac{b^3h}{12}
\]

---

Table A-20 to A-22

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels

*The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm (0.7 to 1.3 in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.*


<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNS No.</strong></td>
<td><strong>SAE and/or AISI No.</strong></td>
<td><strong>Processing</strong></td>
<td><strong>Tensile Strength, MPa (ksi)</strong></td>
<td><strong>Yield Strength, MPa (ksi)</strong></td>
<td><strong>Elongation in 2 in, %</strong></td>
<td><strong>Reduction in Area, %</strong></td>
<td><strong>Brinell Hardness</strong></td>
</tr>
<tr>
<td>G10060</td>
<td>1006</td>
<td>HR</td>
<td>300 (43)</td>
<td>170 (24)</td>
<td>30</td>
<td>55</td>
<td>86</td>
</tr>
<tr>
<td>G10100</td>
<td>1010</td>
<td>CD</td>
<td>330 (48)</td>
<td>280 (41)</td>
<td>20</td>
<td>45</td>
<td>95</td>
</tr>
<tr>
<td>G10100</td>
<td>1010</td>
<td>HR</td>
<td>320 (47)</td>
<td>180 (26)</td>
<td>28</td>
<td>50</td>
<td>95</td>
</tr>
</tbody>
</table>