UTOPIA Finite Different Lattice Boltzmann Method for Simulation Natural Convection Heat Transfer from a Heated Concentric Annulus Cylinder

O. Shahrul Azmir
Thermo-fluid Department, Universiti Teknologi Malaysia, UTM Skudai, Johor, Malaysia

C. S. Nor Azwadi
Thermo-fluid Department, Universiti Teknologi Malaysia, UTM Skudai, Johor, Malaysia
E-mail: azwadi@fkm.utm.my
Tel: +60-7-5534718; Fax: +60-7-5566169

Abstract

This paper presents numerical study of flow behavior from a heated concentric annulus cylinder at various Rayleigh number $Ra$, Prandtl number $Pr$ and aspect ratio of the outer and inner cylinders. The finite different lattice Boltzmann method (FDLBM) numerical scheme is proposed to improve the computational efficiency and numerical stability of the conventional method. The proposed FELBM applied UTOPIA approach (third order accuracy in space) to study the temperature distribution and the vortex formation in the annulus. The comparison of the flow pattern and temperature distribution for every case via streamline, vortices and temperature distribution contour with published paper in literature were carried out for the validation purposes. Current investigation concluded that the UTOPIA FDLBM is an efficient approach for the current problem in hand and good agreement with the benchmark solution.

Keywords: UTOPIA, finite difference, lattice Boltzmann, natural convection, concentric cylinder

1. Introduction

Natural convection in an annulus concentric cylinder has been extensively investigated due to the variety of technical applications and practicality such as heat transfer in heat exchanger (Inamuro, Yoshino and Ogino, 1995), thermal storage system, electrical transmission cables (Nor Azwadi and Tanahashi, 2007), etc. Among the problems related to natural convection, many researchers focused their investigation on the heat transfer and fluid flow behavior from differentially heated walls in a square or cubic cavity (Rohde, Kandhai, Derksen and Akker, 2003)(Jami, Mezrhab, Bouzidi and Lallemand, 2006). However, the heat transfer mechanism and fluid flow behavior in a concentric annulus cylinder are strongly depended on the aspect ratio which is defined as a ratio of the diameter of the outer to the inner cylinder. The temperature different between the heated inner cylinder and cold outer cylinder contributes the density gradient and circulate the fluid in the annulus (Nor Azwadi and Tanahashi, 2007). The next important dimensionless parameters are the Rayleigh and Prandtl numbers, which affect the heat transfer mechanism, the flow pattern and the stability of the transitions of flow in the system.
The lattice Boltzmann method (LBM), a mesoscale numerical method that will be used in present study, is an alternative computational approach to predict wide range of macroscale heat transfer and fluid flow behaviour (Shan, 1997). LBM has been used for nearly a decade due to its simplicity and easy implementation. Due to its ability to incorporate particles interactions at the microscopic level, LBM fits for simulation the behavior of complex flow system (He, Zou, Lou and Dembo, 1997), turbulent (Jonas, Chopard, Succi and Toschi, 2000), multiphase (McNamara and Alder, 1993), and others (Mezrhab, Jami, Abid, Bouzidi and Lallemand, 2006) (Azwadi and Tanahashi, 2006).

In the finite difference formulation of LBM, the governing equations are discretized in the space and time and solved using third order accuracy scheme (UTOPIA). There are many other discretization methods proposed earlier in the literature. However, due to the hyperbolic nature of the Boltzmann equation, the UTOPIA scheme is the best choice due to its high accuracy and flexible boundary treatment (Nor Azwadi, 2009).

This paper is organized as follows. Section two briefly describes the formulation of lattice Boltzmann method. The combination of UTOPIA finite difference lattice Boltzmann (FDLBM) simulation is discussed in section three. The computational results will be discussed by comparing with the available results in the literature and shown in section four. The final section concludes current study.

2. Numerical Method

2.1. Lattice Boltzmann Method

The governing equation of two dimensions lattice Boltzmann scheme, discretised in space and time, can be represented as follow

$$\frac{\partial f_i}{\partial t} + c_i \cdot \nabla f_i = \Omega(g_i) + F$$

(1)

$$\frac{\partial g_i}{\partial t} + c_i \cdot \nabla g_i = \Omega(f_i)$$

(2)

where $f_i$ is the particle distribution function with constant velocity $c_i$ at position $x$ and $\Omega$ is the collision integral. Distribution function $f_i$ and $g_i$ are used to calculate the velocity and temperature fields and $F$ is the external force. However, we can replace the collision integral in Eqs. (1) and (2) with the most well accepted BGK collision model version due to its simplicity and efficiency of the model (Nor Azwadi and Tanahashi, 2006). The equations that represent this model can be written as

$$\Omega(f_i) = -\frac{1}{\tau_f}[f_i - f_i^{eq}]$$

(3)

$$\Omega(g_i) = -\frac{1}{\tau_g}[g_i - g_i^{eq}]$$

(4)

where $f_i^{eq}$ and $g_i^{eq}$ are the equilibrium distribution functions. $\tau_f$ and $\tau_g$ are the time to achieve the equilibrium condition during the collision process and often called as the time relaxation for momentum and energy respectively. The BGK collision model describes that $1/\tau$ of the non-equilibrium distribution function relaxes to the equilibrium state within time $\tau$ during every collision process. By substituting Eqs. (3) and (4) into Eqs. (1) and (2), the so-called lattice Boltzmann BGK equation is obtained and can be written as follow

$$\frac{\partial f_i}{\partial t} + c_i \cdot \nabla f_i = -\frac{1}{\tau_f}[f_i - f_i^{eq}] + F$$

(5)

$$\frac{\partial g_i}{\partial t} + c_i \cdot \nabla g_i = -\frac{1}{\tau_g}[g_i - g_i^{eq}]$$

(6)

Eqs. (5) and (6) describe two main processes in the LBM formulation. The left hand side of each equation refers to the propagation of the distribution function to the neighbor nodes in the
direction of its velocity and the right hand side describes the collision process of the particles distribution function. The general form of the lattice velocity model is expressed as \(DnQm\) where \(D\) represents the spatial dimension and \(Q\) is the number of connection (lattice velocity) at each node (Jonas, Chopard, Succi and Toschi, 2006).

In present study, the D2Q9 and D2Q4 are chosen due to its efficiency and flexibility for boundary treatment. The microscopic velocity components for this model can be simplified as

\[
c_i = \begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix} \quad \text{for D2Q9}
\]

\[
c_i = \begin{bmatrix} -1 & 1 \end{bmatrix} \quad \text{for D2Q4}
\]

\(c_i\) in LBM is set up so that in each time step \(\Delta t\), every distribution function propagates in a distance of lattice node spacing \(\Delta x\). This will ensure that the distribution function arrives exactly at the lattice node after \(\Delta t\) and collide simultaneously.

The equilibrium distribution function of the D2Q9 and D2Q4 models are expressed as follow (Nor Azwadi and Tanahashi, 2006)

\[
f_i^{eq} = \rho \omega_i \left[ 1 + 3 (c_i \cdot u) + \frac{9}{2} (c_i \cdot u)^2 - \frac{9}{2} u^2 \right]
\]

\[
g_i^{eq} = \frac{1}{4} \rho T \left[ 1 + 3 (c_i \cdot u) \right]
\]

respectively and the value of weights are depend on the microscopic velocity direction.

The macroscopic density \(\rho\), velocity \(u\) and temperature \(T\) are defined from the velocity moments of distribution function and can be calculated as follow

\[
\rho = \sum_{i=0}^{q} f_i
\]

\[
\rho u = \sum_{i=0}^{q} c_i f_i
\]

\[
\rho T = \sum_{i=0}^{q} c_i g_i
\]

and the pressure is related to density via \(p = \frac{1}{\sqrt{\rho}}\) where in lattice Boltzmann formulation, \(c_\sigma\) is taken as \(\frac{1}{\sqrt{3}}\).

The macroscopic equation can be obtained via the Chapmann-Enskog expansion. The details of the derivation can be seen in (Azwadi and Tanahashi, 2006) and will not be shown here. By comparing the obtained macroscopic equations with the Navier-Stoke equation derived from Newton’s law, the time relaxations \(\tau\) can be related to fluid viscosity \(\nu\) and thermal diffusivity \(X\) as follow

\[
\tau_f = 3 \nu
\]

\[
\tau_p = X
\]

respectively.

### 2.2. Finite Difference Lattice Boltzmann Method

In the finite difference formulation of LBM, the governing equations are further decrize in phase space and time and solve using one of the several available methods in finite difference numerical scheme. In present study, we choose explicit discretisation in time as follow

\[
f_i^{n+1} - f_i^n + \Delta t [c_i \cdot \nabla f_i] = \frac{\Delta t}{c_s} [(f_i - f_i^{eq})] + \Delta t F
\]

The most common finite difference discretisation scheme that normally used by the researchers is the second order accuracy of upwind scheme. However, in this paper, we applied the third order
The accuracy of upwind scheme (UTOPIA) for spatial discretisation in order to generate more accurate results. The UTOPIA scheme in $x$- and $y$- directions are expressed as follow:

$$
\left( \frac{\partial f(x)}{\partial x} \right)_i = \begin{cases} 
\frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x} & c_x \neq 0 \\
-\frac{f(x_{i-1}) - f(x_{i+1})}{2\Delta x} & c_x = 0
\end{cases}
$$

$$
\left( \frac{\partial f(y)}{\partial y} \right)_i = \begin{cases} 
\frac{f(y_{i+1}) - f(y_{i-1})}{2\Delta y} & c_y \neq 0 \\
-\frac{f(y_{i-1}) - f(y_{i+1})}{2\Delta y} & c_y = 0
\end{cases}
$$

(16)

(17)

3.0. Simulation Results

In this section, we will demonstrate the capability of finite different lattice Boltzmann method in predicting fluid flow and heat transfer problem. For the sake of code validation, we will consider the problem of heat transfer from a heated concentric annulus cylinder and the comparison of the results will be carried out with the benchmark solution provided by Shi, 2006.

3.1. Code’s Validation

Figure 1 shows the geometry of the heated annulus cylinder problem considered in the present study.

Figure 1: Natural convection heat transfer in concentric annulus cylinder

where $g$ is the gravitational acceleration and $T_c$ and $T_h$ are the constant temperatures of the outer and inner cylinder respectively. The ratio of the radius of outer cylinder to the inner cylinder is set at 2.6 in all of simulation in order as in Shi, 2006. Two non-dimensionless parameters; the Prandtl number $Pr$, and the Rayleigh number $Ra$, for heat transfer behaviour of this problem are characterized and defined as

$$
Pr = \frac{\nu}{\kappa}
$$

$$
Ra = \frac{\beta (T_h - T_c) \kappa g}{\nu \kappa}
$$

(18)

(19)
where $\nu$ is the fluid kinematic viscosity, $\chi$ is the thermal diffusivity and $L = r_0 - r_i$ is the characteristic length.

In our numerical simulation, the Prandtl numbers are set at 0.716, 0.717, 0.717, 0.718 and 0.718 for Rayleigh numbers $2.38 \times 10^3$, $9.50 \times 10^3$, $3.20 \times 10^4$, $6.19 \times 10^4$ and $1.02 \times 10^5$ respectively.

**Figure 2**: Plots of isotherms for different Rayleigh and Prandtl numbers obtained from the LBM

\begin{itemize}
  \item (a) $Ra = 2.38 \times 10^3$, $Pr = 0.716$
  \item $Ra = 9.5 \times 10^3$, $Pr = 0.717$
  \item $Ra = 3.20 \times 10^4$, $Pr = 0.717$
  \item $Ra = 6.19 \times 10^4$, $Pr = 0.718$
  \item $Ra = 1.02 \times 10^5$, $Pr = 0.718$
\end{itemize}
Figure 3: Plots of isotherms for different Rayleigh and Prandtl numbers obtained by Shi, 2006

(a) $Ra = 2.38 \times 10^3$, $Pr = 0.716$

(b) $Ra = 9.5 \times 10^3$, $Pr = 0.717$

(c) $Ra = 3.20 \times 10^4$, $Pr = 0.717$

(d) $Ra = 6.19 \times 10^4$, $Pr = 0.718$

(e) $Ra = 1.02 \times 10^5$, $Pr = 0.718$

Figure 2 shows the plots of isotherms for different Rayleigh and Prandtl numbers gathered from the LBM simulation while in figure 3 represent the same numerical simulation for validation purposes from established results by Shi, 2006. As it can be seen from the figure 2 and 3, the results are almost identical and qualitatively good agreement with the benchmark solution.

3.2. Flow and Temperature Fields at various Aspect Ratios

In this section, we will demonstrate the computational results to discuss the effects of the aspect ratio on the heat transfer mechanism and the fluid flow behaviour in the annulus. Figure 4 shows the plots of isotherms (right) and streamline (left) for various values of aspect ratio. In present study, we fix the value of Rayleigh and Prandtl numbers at $3.20 \times 10^4$ and 0.717 respectively. The sizes of mesh are set at $124 \times 124$, $114 \times 114$ and $112 \times 112$ for aspect ratio 2.6, 3.6, 4.6 and 5.0 respectively.
As can be seen from figure 4, the simulated results at all values of aspect ratio showing the desorted line above the heated inner cylinder due to the buoyancy effect. This indicates that the convection is the dominant mode heat transfer mechanism at this condition. Due to the convection process, the isotherms move upward and larger plumes exits around the top of the outer cylinder indicating the present of strong thermal gradient at this region.

The plots of streamlines shows the main flow is formed at the upper half of the annulus. This can be seen where the center of the main vortex is located at this region. By increasing the aspect ratio, the size of vortex also increases and the isotherms are distored more due to the stronger convection effect, leading to stable stratification of the isotherms. The vortex slightly move upward due to higher rotational flow resulted from more spacing between the two cylinders. The plots of temperature contour for every simulated aspect ratio are shown in figure 5.
4. Conclusion
In this paper, a finite difference lattice Boltzmann method have been applied to predict the heat transfer from a heated annulus cylinder with various aspect ratio at fixed Rayleigh number of $3.20 \times 10^4$. The spatial discretisation has been carried out using third order accuracy to ensure the validity of the computed results. All the simulated results agree well with the benchmark solution reported in the previous studies. Extension to three dimensions and application to more complex geometries will be our future investigation.

References