Chapter 6

Stability Analysis of Disc Brake Model

6.1 Introduction
The motivation of present chapter is to determine stability of the disc brake mostly using the complex eigenvalue analysis. Experimental results obtained by James (2003) are utilised for comparison and validation. The first section is devoted to determining kinetic friction coefficients, which then will be used throughout the analysis. A simple mathematical model of a disc brake is described to form a basic formula of kinetic friction coefficient. The complex eigenvalue analysis is performed for different friction characteristics including friction damping for the real contact interface model and the perfect contact interface model of the friction material. Then the analysis is performed for the validated FE model considering the effect of wear. Next stability analysis of different FE models (described in section 5.2.3) is conducted to see which one produces numerical results that are close to experimental results of squeal events. Unstable modes are also visualised and compared with the squeal experiments. The next section is focused on the dynamic transient analysis in which previously developed full disc brake model is reduced to only the disc and two pads. Predicted unstable frequencies then will be compared with the results of the complex eigenvalue analysis. The squeal frequency range of interest in this investigation is between 1 kHz and 8 kHz, which the mode shapes are characterised up to 7 nodal diameters.

6.2 Experimental Results

6.2.1 Determination of Kinetic Friction Coefficient
It is essential to determine coefficient of friction that is generated between the disc and the pad contact interface. It has been known that coefficient of friction would be changed under different braking operating conditions such as brake-line pressure, disc speed, temperature and other factors. Braking torque at the contact interface can be predicted by a simple mathematical model as shown in figure 6.0. From figure 6.0a, braking torque can be calculated as follows:
\[ T = F_{\text{friction}} r_{\text{eff}} \]  \hspace{1cm} (6.0)

where \( F_{\text{friction}} \) is the friction force generated at the contact interface and \( r_{\text{eff}} \) is the effective pad radius. However friction force is dependent upon normal force \( F_{\text{normal}} \) and friction coefficient \( \mu \), which is derived as below:

\[ F_{\text{friction}} = \mu F_{\text{normal}} \]  \hspace{1cm} (6.1)

Normal force \( F_{\text{normal}} \) can be determined based on brake-line pressure \( p \) applied onto top of the piston, as shown in figure 6.0b, which is given in the following equation:

\[ F_{\text{normal}} = p A_{\text{piston}} \]  \hspace{1cm} (6.2)

Now by substituting equation (6.2) into equations (6.1) and then (6.0), braking torque can be calculated as follows:

\[ T = \mu p A_{\text{piston}} r_{\text{eff}} \]  \hspace{1cm} (6.3)

For a disc brake system there is a pair of brake pads, thus the total brake torque is:

\[ T = 2 \mu p A_{\text{piston}} r_{\text{eff}} \]  \hspace{1cm} (6.4)

Since the braking torque output \( T \) can be obtained experimentally, and the cross sectional area of the piston in contact with the braking fluid \( A_{\text{piston}} \), brake-line pressure \( p \) and pad effective radius \( r_{\text{eff}} \) are all known parameters, then coefficient of friction can be calculated. Thus equation (6.4) now becomes:

\[ \mu = \frac{T}{2 p A_{\text{piston}} r_{\text{eff}}} \]  \hspace{1cm} (6.5)
In order to calculate kinetic friction coefficients, data of those parameters are taken from (James, 2003). Table 6.0 shows values of those parameters for different brake squeal tests and coefficients of friction that are calculated using equation (6.5).

Table 6.0: Operating conditions of squeal test (James, 2003)

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Torque, $T$ (Nm)</th>
<th>Pressure, $p$ (MPa)</th>
<th>Speed, $\Omega$ (rad/s)</th>
<th>Friction Coefficient, $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T01</td>
<td>77.78</td>
<td>0.3341</td>
<td>3.22</td>
<td>0.458</td>
</tr>
<tr>
<td>T02</td>
<td>74.41</td>
<td>0.3230</td>
<td>3.23</td>
<td>0.453</td>
</tr>
<tr>
<td>T03</td>
<td>37.51</td>
<td>0.2087</td>
<td>6.29</td>
<td>0.354</td>
</tr>
<tr>
<td>T04</td>
<td>41.10</td>
<td>0.2222</td>
<td>6.30</td>
<td>0.364</td>
</tr>
<tr>
<td>T05</td>
<td>225.73</td>
<td>0.8504</td>
<td>6.29</td>
<td>0.522</td>
</tr>
<tr>
<td>T06</td>
<td>210.93</td>
<td>0.8329</td>
<td>6.32</td>
<td>0.498</td>
</tr>
<tr>
<td>T09</td>
<td>52.71</td>
<td>0.2471</td>
<td>15.47</td>
<td>0.420</td>
</tr>
<tr>
<td>T12</td>
<td>24.28</td>
<td>0.1620</td>
<td>15.49</td>
<td>0.294</td>
</tr>
<tr>
<td>T19</td>
<td>37.51</td>
<td>0.1554</td>
<td>25.57</td>
<td>0.475</td>
</tr>
<tr>
<td>T25</td>
<td>87.43</td>
<td>0.4016</td>
<td>3.234</td>
<td>0.428</td>
</tr>
<tr>
<td>T27</td>
<td>182.26</td>
<td>0.6581</td>
<td>3.23</td>
<td>0.545</td>
</tr>
<tr>
<td>T29</td>
<td>18.32</td>
<td>0.1221</td>
<td>100.30</td>
<td>0.295</td>
</tr>
<tr>
<td>T35</td>
<td>35.15</td>
<td>0.2304</td>
<td>61.48</td>
<td>0.300</td>
</tr>
<tr>
<td>T36</td>
<td>36.38</td>
<td>0.2279</td>
<td>61.47</td>
<td>0.314</td>
</tr>
<tr>
<td>T39</td>
<td>34.55</td>
<td>0.1748</td>
<td>41.02</td>
<td>0.389</td>
</tr>
<tr>
<td>T40</td>
<td>41.26</td>
<td>0.1976</td>
<td>41.04</td>
<td>0.411</td>
</tr>
<tr>
<td>T43</td>
<td>28.73</td>
<td>0.1720</td>
<td>3.22</td>
<td>0.329</td>
</tr>
<tr>
<td>T44</td>
<td>83.90</td>
<td>0.3401</td>
<td>3.21</td>
<td>0.485</td>
</tr>
<tr>
<td>T45</td>
<td>231.60</td>
<td>0.8062</td>
<td>3.22</td>
<td>0.565</td>
</tr>
<tr>
<td>T46</td>
<td>30.01</td>
<td>0.1778</td>
<td>15.51</td>
<td>0.332</td>
</tr>
<tr>
<td>T47</td>
<td>95.46</td>
<td>0.3780</td>
<td>3.278</td>
<td>0.497</td>
</tr>
</tbody>
</table>
It is also worthwhile to see how friction coefficients vary with sliding speeds and brake-line pressures. Based on the results described in Table 6.0, the relationship between friction coefficients, sliding speeds and brake-line pressures can be found. Figure 6.1 seems to suggest that friction coefficient decreases as sliding speed increases. This may lead to a term called negative $\mu - \nu$ slope and consequently will produce negative damping. This characteristic also may increase instability in the system. In addition an increase in brake-line pressure is seen also to increase friction coefficient. The lowest friction coefficient is 0.295, which is generated at brake-line pressure of 0.122 MPa and the highest friction of 0.565 is generated at brake-line pressure of 0.806 MPa. All the information obtained in this section will be used for subsequent analysis.

Figure 6.1: Relationship between friction coefficient and sliding speed (top) and brake-line pressure (bottom)
6.2.2 Squeal Results

Squeal results obtained by James (2003) are fully utilised. Although three different brake discs were studied, only the solid disc of Mercedes brake is considered since here the FE model is developed for it. In the squeal tests, which were conducted under a series of operating conditions as listed in Table 6.0, squeal frequencies up to 8 kHz were captured and it was found that all of the squeal frequencies were dominated by out-of-plane vibration modes. There was no single in-plane vibration mode captured during squeal events. This might be due to the equipment and tools available were not able to capture such a vibration mode. In Table 6.1, it shows that squeal events are most often generated at frequencies of 4 kHz and 7.5 kHz, which were characterised by 5 and 7 nodal diameters respectively. The less appearance of squeal event is around 1.8 kHz, which was formed by 3 nodal diameters.

Table 6.1: Squeal Test Results (James, 2003)

<table>
<thead>
<tr>
<th>Nodal Diameters</th>
<th>Mean Squeal Frequency (Hz)</th>
<th>No. of Appearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1776.3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>2786.9</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>4050.9</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>6451.5</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>7531.8</td>
<td>7</td>
</tr>
</tbody>
</table>

From figure 6.2, it can be seen that most squeal events occurred at low brake-line pressure between 0.1 to 0.4 MPa. There was only one squeal event at brake-line pressure of around 0.68 MPa, in comparison with four squeal events at above 0.8 MPa. However, there is no squeal event recorded at brake-line pressure between 0.4 to 0.6 MPa. These squeal results are useful for subsequent analyses by which comparison and correlation can be made in order to determine improvement of using the proposed methodology.
6.3 Complex Eigenvalue Analysis

This section focuses on prediction of unstable frequencies using the complex eigenvalue analysis and describes disc brake vibration characteristics at a system level. Prediction of the system instability is made at frequencies similar to those of squeal tests conducted above. The first part in this section is to investigate effect of several friction characteristics and damping on unstable frequencies for the real and the perfect contact interface. From the results, conclusion can be drawn by which friction features and type of contact interface can produce more accurate and reliable prediction. The next parts are to study effects of using different types of modelling and of wear on unstable frequencies.

6.3.1 Effects of Friction Characteristics and Friction Damping

It is already known that friction is the main parameter for squeal to emanate. In the past, squeal was believed to be due to the effect of negative $\mu - \nu$ slope alone (Mills, 1938; Sinclair, 1955 and Fosberry and Holubecki, 1955 and 1961). However, Spurr (1961-1962), Earles and Soar (1971), North (1972) and Millner (1978) proved that a constant friction coefficient also could generate squeal. This observation was also supported in recent investigation by Eriksson and Jacobson (2001) where they found there was no correlation of squeal with the negative $\mu - \nu$ slope in their squeal experiments. However, negative $\mu - \nu$ slope has demonstrated to increase tendency of system instability in the complex eigenvalue analysis investigated by Kung et al (2003) and Bajer et al (2004). Thus, this type of mechanism is still significant in
studying brake squeal. It is a standard practice in modelling a brakes system that material damping of brakes components is neglected. This is due to the difficulty in quantifying the damping value in experiments and also to simplify the disc brake squeal problem as well as to observe squeal behaviour without it. However, damping can also be formed due to friction, which has been included by Kung et al (2003) in their study. This damping terms exist due to two effects: the first is caused by friction forces in slip direction and the second effect is caused by a velocity – dependent friction coefficient. In this section several friction characteristics and the effect of friction damping are investigated for the real and the perfect contact interfaces.

For the following work, the complex eigenvalue analysis is performed for all the operating conditions as stated in Table 6.0. This is to make sure that comparison in terms of unstable frequencies can be made between predicted and measured data for a certain range of brake-line pressure as illustrated in figure 6.2.

6.3.1.1 Constant friction coefficient

In this section, constant friction coefficient is used when static friction coefficient is the same as kinetic friction coefficient. It means that friction coefficient is the same when the disc is in stationary and in rotational stages. In assuming a constant coefficient, friction coefficients calculated in Table 6.0 are used for both stationary (static coefficient) and rotational (kinetic coefficient) discs.

For the perfect contact interface, predicted unstable frequencies for brake-line pressure between 0.0 ~ 1.0 MPa are shown in figure 6.3a. It can be seen from the figure that unstable frequencies around 2 kHz and 3 kHz are not predicted. While for the real contact interface the prediction, as shown in figure 6.3b, seems to show that the number of unstable frequencies is fewer than those predicted for the perfect contact interface. There are also a number of unstable frequencies missing particularly in the frequency range of 4 ~ 7 kHz at brake-line pressure range of 0.0 ~ 0.4 MPa. The above results indicate that neither the perfect nor the real contact interfaces can predict unstable frequencies very well with the experimental results when a constant friction coefficient is adopted.
6.3.1.2 Constant friction coefficient with friction damping

In this analysis, the effect of friction damping is considered along with a constant friction coefficient. The friction damping term in ABAQUS can be activated by specifying FRICTION DAMPING = YES in the input file. This damping is caused by the friction forces that stabilising vibration along the contact surface in direction perpendicular to the slip direction (ABAQUS, Inc, 2003). This effect can be seen in the third term of equation (3.13) in chapter 3. The friction damping may suppress some of unstable modes.

It can be seen that for the perfect contact interface unstable frequencies shown in Figure 6.4a are predicted differently from those in Figure 6.3(a). Some of the
unstable frequencies of Figure 6.3 now become stabilised but some new unstable frequencies have emerged. Using the combination of constant friction coefficient and friction damping, prediction of unstable frequencies is much better than considering constant friction coefficient alone. All the squeal frequencies are well predicted at the brake-line pressure of between 0.6 ~ 1.0 MPa. However, there are still a number of unstable frequencies missing in the range of 0.0 ~ 0.4 MPa. For the real contact interface a slight improvement is achieved, as illustrated in Figure 6.4b, compared to those predicted in Figure 6.3b. Both contact interface models agree well with the experimental data in the range of 0.6 ~ 0.8 MPa of the brake-line pressure. Even though there is an improvement in the prediction using a combination of constant friction coefficient and friction damping, it is thought that the overall prediction is still not sufficiently good, compared with the experimental results.

![Figure 6.4: Prediction of unstable frequencies with the effects of constant friction coefficient and friction damping](image)

a) Perfect Contact Interface
b) Real Contact Interface

Figure 6.4 (cont’d): Prediction of unstable frequencies with the effects of constant friction coefficient and friction damping

6.3.1.3 Negative $\mu$-$\nu$ slope

In this section, a velocity-dependent friction coefficient is used. If the friction coefficient decreases with velocity (disc speed) the effect is destabilising and thus more unstable frequencies will emerge. This effect is known as negative damping in the literature. Negative damping effect can be seen in the second term of equation (3.13). In order to activate this effect in ABAQUS, different values of friction coefficient are used. The static friction coefficient is set at $\mu = 0.7$ for all the operating conditions while the smaller kinetic friction coefficient is taken from Table 6.0.

From figure 6.5a in which the perfect contact interface is used, it is seen that more unstable frequencies are predicted than those previous two results. It seems that when considering negative damping effect, predicted unstable frequencies are much closer to those obtained in the squeal experiments (figure 6.2). However, there are still a number of unstable frequencies missing particularly around 2 kHz for the whole range of the brake-line pressure. For the real contact interface model predicted unstable frequencies, as illustrated in figure 6.5b, are much fewer than those predicted using the perfect contact interface. The real contact interface seems to lead to a better correlation with the experimental results around 2 kHz than the perfect contact interface. However, there are also some unstable frequencies missing, especially at frequencies of at 3 kHz and 7 kHz and the brake-line pressure of below
0.4 MPa. Nevertheless, both contact interface models agreed well with the experimental data in the pressure range of 0.6 ~ 0.8 MPa. By considering the negative damping effect, the overall prediction for the perfect and the real contact interface models give much better agreement than the previous two predicted results. However, it will be shown that the agreement with experimental results can still be improved.

![Graph showing prediction of unstable frequencies under negative μ − ν slope characteristic](image)

**Figure 6.5:** Prediction of unstable frequencies under negative $\mu - \nu$ slope characteristic

6.3.1.4 Negative $\mu - \nu$ slope with friction damping

The last consideration in this study is to combine the effect of negative damping with friction damping. Explanations on inclusion of negative damping and friction damping in ABAQUS input file are described in the previous section. For the perfect
contact interface model, it can be seen from figure 6.6a that most unstable frequencies now are predicted much closer to those obtained in the squeal experiments. There are only four unstable frequencies missing at the frequency of 3 kHz under the brake-line pressure of around 0.2 MPa. This represents 85 percent accuracy against the experimental results. For the real contact interface model, good agreement with the experimental data is achieved, as depicted in figure 6.6b. There are only three unstable frequencies missing now, in which two of them are located at 3 kHz (near 0.2 MPa) and the other is located at about 6 kHz (around 0.1 MPa). This represents about 89 percents accuracy in prediction. Based on the above results it can be said that considering negative $\mu - \nu$ slope with friction damping give the best agreement with the experimental results among the four regimes of friction behaviour studied.

Figure 6.6: Prediction of unstable frequencies with the effects of negative damping and friction damping
It is also significant to see which contact interface model can produce better prediction in terms of accuracy and reliability. The perfect contact interface model is always predicting more unstable frequencies than the real contact interface model. This agrees well with the finding obtained by Soom et al (2003) that a smooth surface of friction material tended to increase the number of squeal than a rough surface. This seems to show that using the real contact interface, over-prediction as mentioned by Kung et al (2003), can be reduced and this is proved in the results. In turn, it could provide much better reliability in unstable frequency prediction. Furthermore, in terms of accuracy, the real contact interface model produces much better prediction than the perfect contact interface model by 4 percents. Therefore, the real contact interface model, as has been a main topic of the proposed methodology, is proved to produce higher reliability and accuracy in prediction than the perfect contact interface and hence is chosen for subsequent stability analysis.

6.3.2 Unstable Complex Modes of Disc Brake Model

It is important that validations of the prediction results are not only based on the unstable frequency alone but also in terms of its unstable mode shapes. Figure 6.7 shows predicted unstable frequencies over brake-line pressure ranging from 0.0 ~ 1.0 MPa. The positive real parts indicate tendency of squeal to occur. This prediction result is based on the real contact interface model. From the results, it is shown that the disc brake model become unstable at frequencies of around 1.8 kHz, 2.2 kHz, 4.0 kHz, 6.5 kHz, 7.0 kHz, 7.5 kHz and 8.0 kHz. This prediction seems to cover all squeal events observed in the experiments as described in Table 6.1. Predicted complex eigenvalues and associated modes of some test cases described in Table 6.0 are presented in the subsequent figures of this section, for the sake of validation.
Figure 6.7: Unstable frequencies predicted by the complex eigenvalue analysis over operational range of brake

In order to validate the first squeal frequency, operating conditions of T43 as described in Table 6.0 is used. Based on this operating condition, squeal frequency in the experiments was found at 1892 Hz, which is characterised by 3 nodal diameters. For the complex eigenvalue analysis, unstable frequency (real parts) is predicted at 1521 Hz with an instability measurement equal to + 0.8, which is also characterised by 3 nodal diameters illustrated in figure 6.8a. This unstable frequency is a bit closer with natural frequency of the disc at free-free boundary condition and assembly described in chapter 4. For this instability, the piston (bottom) and the finger (upper) pads are experiencing first twisting mode as shown in figure 6.8b. Contrary to free-free boundary condition of modal analysis of the brake pad, this mode is found at a frequency less than its natural frequency.

Figure 6.8: Unstable modes at frequency of 1521 Hz
The calliper, as shown in figure 6.9a, is seen to have twisting mode at its arms. The carrier seems to experience second bending mode at its bridge from top view with the torque members are also distorted as shown in figure 6.9b. By looking at the rest of disc brake components, it is suggested that the piston and the guide pins and bolts are undergoing a rigid body mode. This can be seen from deformation of the components as shown in figure 6.9. Comparing to their natural frequencies of free-free boundary condition explained in chapter 4, none of these components except the carrier has any natural frequencies that are close to this unstable frequency. The carrier has a close frequency but with a different mode from the unstable mode shape as shown in figures 4.4a and 6.9b respectively. This seems to indicate that the closeness in natural frequency of these components to the disc has somewhat nothing to do with the emergence of instability of the disc brake model.
The second squeal event found in the experiment was at frequency of 2954 Hz in which the disc undergoes its 4 nodal diameters of deformation. This squeal frequency was observed at operating conditions of T40 (refer to Table 6.0). The prediction result shows that using a similar operating condition unstable frequency is found at 2847 Hz with the positive real parts of +2.2. At this unstable frequency, the disc is characterised by 4 nodal diameters illustrated in figure 6.10a. This describes that a good agreement is reached in terms of mode shape and its frequency of the disc. This unstable frequency seems slightly less than its natural frequency of free-free boundary condition. The finger pad (upper) and the piston pad (bottom) as shown in figure 6.10b illustrate that they are first bending mode. This unstable frequency is still less than its natural frequency in similar type of mode shapes found in previously chapter.

Considering the rest of the disc brake components, it is shown in figure 6.11 that the calliper experiences significant bending at its fingers and they seem to move away from each other while its arms exhibit a small degree of twisting motion. This unstable mode shape is deformed differently to its normal mode shape with closer frequency. The carrier bridge seems to deform in first bending mode and the unstable frequency is less than its natural frequency of free-free boundary condition. The piston and the guide pin and bolt are experiencing a rigid body mode.
For the operating conditions of T03, squeal was found at frequency of 4275 Hz, in which the disc vibrated in 5 nodal diameters. In simulation, an unstable frequency of 4340 Hz with instability measurement of +17.4 is predicted with similar disc vibration behaviour observed in the experiments. Figure 6.12a shows the deformation of the disc model in 5 nodal diameters. Again, this unstable frequency is closer to its natural frequency either in free-free boundary condition or in assembly stage. Comparing the frequency and its mode, it is suggested that a good correlation is achieved between predicted and experimental results. From figure 6.12b, it shows that both pads deform in first twisting mode. This is similar to the normal mode shape found in a closer natural frequency as shown in figure 4.2.

At this operating condition of a braking application, the calliper seems to experience bending displacement at it arms as illustrated in figure 6.13a and the carrier looks to deform at its bridge as first bending mode with the torque members move away from each other. The piston and the guide pin as shown in figures 6.13c and 6.13d are deformed as a rigid body mode. There is no apparent deformation found in these
components. It seems to suggest that these components did not have any contribution towards instability of the disc brake model except the brake pads and the disc itself.

![Unstable modes at frequency of 4340 Hz](image)

**Figure 6.12:** Unstable modes at frequency of 4340 Hz

![Unstable modes of the disc brake components at frequency of 4340 Hz](image)

**Figure 6.13:** Unstable modes of the disc brake components at frequency of 4340 Hz

For 6 nodal diameters of an unstable mode of the disc, operating conditions of T01 are used in which squeal frequency was found at 6599 Hz. This squeal frequency is quite close with simulation where unstable frequency of 6247 Hz is predicted with the positive real parts of +17.2. A similar type of the disc mode, as shown in figure
6.14a, is found and hence shows good agreement between the two results. The brake pads as illustrated in figure 6.14b are deformed in first twisting mode in which the finger (upper) has larger deformation than the piston (bottom) pads. The calliper as shown in figure 6.15a shows that its arms is experiencing twisting and bending modes simultaneously and one of its fingers (bottom) seems to displace towards its housing. The carrier in figure 6.15b shows that it experiences a second bending mode at its bridge from top view. The piston and the guide pin illustrated in figures 6.15c and 6.15d show that they deform in second diametric mode and second bending mode respectively.

Figure 6.14: Unstable modes at frequency of 6247 Hz

Figure 6.15: Unstable modes of disc brake components at frequency of 6247 Hz
For 7 nodal diameters of the disc unstable complex mode, operating conditions of T45 are used in which squeal frequency was found at 7420 Hz while in simulation using the complex eigenvalue analysis, unstable frequency of 7402 Hz is predicted with instability measurement of +58.8. Unstable mode shape of the disc depicted in figure 6.16a shows that it deforms in 7 nodal diameters similar to the mode shape found in the experiments. For this instability, the brake pads are undergoing first twisting mode as illustrated in figure 6.16b for the piston (bottom) and the finger pads (upper). The calliper (figure 6.17a) shows that its arms are deformed in bending mode while its fingers deform inside and outside from its parallel location. The carrier as shown in figure 6.17b illustrates that its bridge is experiencing second bending mode. For the piston and the guide pins (figures 6.17c and 6.17d), they are deformed in second diametric mode and first bending mode respectively.

Figure 6.16: Unstable modes at frequency of 7402 Hz
There is an in-plane unstable mode predicted in the simulation at frequency of 2242 Hz, which is not detected in the experiments. This might be due to the incapability of the instrument and equipment that have been used in the experiments to detect any in-plane vibrations even if they exist. From the simulation, it shows that at frequency of 2.2 kHz with the positive real part of +14.4, the disc brake becomes unstable where the disc experiences simultaneous radial and circumferential in-plane unstable vibration as shown in figure 6.18a. The piston pad (bottom) depicted in figure 6.18b seems to show that it experiences first bending mode while the finger pad (upper) experiences first bending mode in the radial direction. The calliper, the piston and the guide pin as shown in figure 6.19 show that they are deformed as a rigid body model. The carrier bridge as illustrated in figure 6.19b is deformed as first bending mode.
Figure 6.18: Unstable modes at frequency of 2242 Hz

Figure 6.19: Unstable modes of disc brake components at frequency of 2242 Hz

From the above results it can be concluded that predictions of the disc brake instability are well correlated with the experimental data in terms of unstable frequencies and complex modes based on each case of operating conditions. It is also suggested that most unstable frequencies and its mode shapes of the disc are quite close to natural frequencies and normal mode shapes of disc in free-free boundary condition. This might explain why some previous researchers (Nishiwaki, 1989, and Fieldhouse and Newcomb, 1993) inclined to hypothesise that squeal was generated when its frequencies during braking applications close to its natural frequencies of
the disc. For rest of the disc brake components including the brake pads, the above results seem to suggest that instability of the disc brake did not depend on coupling of the components’ frequencies with the natural frequencies of the disc. It can be seen from above figures that most of the components vibrate with a rigid body motion. Furthermore, those components in an assembly level are vibrated differently than those found in a free-free boundary condition. Therefore, it is suggested that coupling of natural frequencies of these components with the disc natural frequencies did not contribute towards any disc brake squeal.

### 6.3.3 Different Levels of Modelling

In the past, there have been various levels of disc brake modelling for studying disc brake squeal, for example, from a simplistic model (Ripin, 1995) to a very complicated model (Liles, 1989). In this section, three different models, i.e. baseline model, Model A3 and Model A4 as discussed in chapter 5 are simulated. This is performed in order to see any differences in predicting unstable frequencies. Model A1 and Model A2 cannot be used to predict disc brake instability because they use a rigid surface of the disc.

In this comparison, one of the braking applications as stated in Table 6.0 is selected. From this braking application, it was observed that the squeal occurred at frequencies of about 2.1 kHz and 7.5 kHz as shown in figure 6.20. Using the baseline model which considers a complicated/full disc brake model, these unstable frequencies are well covered with more unstable frequencies appearing in the region of 2 kHz and between 6 ~ 8 kHz. For Model A3 in which only a deformable disc and brake pads were modelled, it is shown that only unstable frequency of 2.1 kHz is found while it misses another unstable frequency at 7.5 kHz. Considering a disc brake model without the carrier as proposed in Model A4 seems to suggest that it can predict unstable frequencies similar to the experimental results. However, more unstable frequencies are predicted as opposed to the baseline model and Model A3, particularly in frequency range of 3 ~ 7 kHz. Based on these prediction results, it is suggested that the current model adopted in this work can predict unstable frequencies much better than the two other models as put forward by previous researchers. Furthermore, careful consideration should be taken when using Model A3 as the model misses one of the important unstable frequencies.
6.3.4 Effect of Wear

In previous chapter, it can be seen that contact pressure distributions vary from one wear simulation to another. Hence it is significant to see how those distributions can change instability of the disc brake. It is also important to see how wear (surface topography of the friction material) as explained by Eriksson et al (1999), Jibki et al (2001), Chen et al (2002) and Sherif (2004), could generate fugitive nature of the disc brake squeal. In order to investigate above statements, six different wear simulations are conducted similarly to those simulated in previous chapter.

In this simulation, operating condition of T45 is chosen. It can be seen from figure 6.21 that at this braking application squeal occurred at frequency of about 7.5 kHz. In the simulation, with initial surface topography of the friction material at the piston and finger pads, this unstable frequency can be predicted very well. After undergoing braking application for 10 minutes as described in Wear 1, this unstable frequency remains. Similarly for Wear 2, where the brake remains applied for another 10 minutes, unstable frequency of 7.5 kHz is still unchanged. However, when it comes to another 10 minutes of braking application for Wear 3, this unstable frequency is gone. This is continued for another 10 minutes as illustrated in Wear 4. Interestingly, this unstable frequency is emerging back in Wear 5 but is gone in Wear 6.
From the results it shows that due to wear, surface topography of the friction materials are changing under different braking applications as shown in figure 6.22. This, in turn, generates different contact pressure distributions as shown in chapter 5 and subsequently it affects occurrence of unstable frequencies. Figure 6.21 clearly displays the fugitive nature of unstable frequencies even though similar boundary conditions and operating conditions are imposed to apparently the same disc brake model. This finding seems to support previous observations obtained by the above researchers that surface topography of the contacting bodies play an important role to the appearance and disappearance of squeal. However, further investigations should be carried out if classification on the roughness of the friction material as proposed by Eriksson et al (1999) and determination of squeal index by Sherif (2004) need to be established. Classification on the roughness and determination of squeal index should easily be obtained in the experiments. However, they are difficult to quantify in the simulation. Therefore, they are not investigated in the present work.

Figure 6.21: Prediction of unstable frequencies under the effect of wear
Figure 6.22: Surface topography of the friction material at the piston (left) and finger (right) pads for different wear simulations.
6.4 Dynamic Transient Analysis

In this section, dynamic transient analysis is performed in order to make comparison with the complex eigenvalue analysis. The main objective is to see correlation between the two analysis methods in terms of predicted unstable frequencies. The authors of (D’Souza and Dweib, 1990b and Tworzydlo et al, 1994) performed a linear and nonlinear stability analysis of a three degrees-of-freedom pin-on-disc model using either Routh criterion or complex eigenvalue analysis along with dynamic transient analysis. Contact forces at the pin/disc interface based on experimental data were utilised in (D’Souza and Dweib, 1990b). On the other hand, an extended version of the Oden-Martin friction model was used in (Tworzydlo et al, 1994) to represent properties of the pin/disk interface. D’Souza and Dweib (1990b) found that using Routh criterion the steady state sliding motion became unstable if average value of friction coefficient reached its critical value. The onset of self-excited vibrations was also predicted with the aforementioned critical value in dynamic transient analysis. Tworzydlo et al (1994) reported that the onset of limit-cycle vibration confirmed predictions of the complex eigenvalue analysis. However the above-mentioned authors did not provide the values of unstable frequencies and hence how well the two methods correlate is not very clear.

Hoffmann et al (2003) demonstrated that a good correlation was clearly shown if both analysis methods dealt with a same two degrees-of-freedom linear oscillator model. With more realistic representation of a disc brake, von Wagner et al (2003) used linear and nonlinear multi degrees-of freedom models to predict instability of the system. They reported that frequency of the limit cycle vibration was nearly the same as that predicted in the complex eigenvalue. Mahajan et al [7] ran both complex eigenvalue analysis and transient analysis for a large degree of freedom disc brake model but comparison was not made between them. In a recent study, Massi and Baillet (2005) demonstrated that the dynamic transient analysis could only capture only one of the two unstable frequencies predicted in the complex eigenvalue analysis for a large degree of freedom model of a disc brake. They used two different finite element software packages though, namely, ANSYS® for the complex eigenvalue analysis and in-house finite element software called PLAST3 for the dynamic transient analysis. For the former there were two unstable frequencies of 3.5
kHz and 5.0 kHz found whilst for the latter only one unstable frequency of 3.4 kHz was captured while there was no indication of instability at 5 kHz. Therefore, it is interesting to see correlation between the two analysis methods that use different system models using a single software package.

### 6.4.1 Explicit Dynamic Analysis

The explicit dynamics analysis in ABAQUS uses central difference integration rule together with the use of diagonal lumped element mass matrices. The following finite element equation of motion is solved:

\[
M \ddot{x}^{(t)} = f_{ex}^{(t)} - f_{in}^{(t)}
\]  

(6.6)

At the beginning of the increment, accelerations are computed as follows:

\[
\ddot{x}^{(t)} = M^{-1}(f_{ex}^{(t)} - f_{in}^{(t)})
\]  

(6.7)

where \( \ddot{x} \) is the acceleration vector, \( M \) is the diagonal lumped mass matrix, \( f_{ex} \) is the applied load vector and \( f_{in} \) is the internal force vector. The superscript \( t \) refers to the time increment.

The velocity and displacement of the body are given in the following equations:

\[
\begin{align*}
\dot{x}^{(t+0.5\Delta t)} &= \dot{x}^{(t-0.5\Delta t)} + \frac{\Delta t^{(t+0.5\Delta t)} + \Delta t^{(t)}}{2} x^{(t)} \\
\dot{x}^{(t+\Delta t)} &= \dot{x}^{(t)} + \Delta t^{(t+\Delta t)} x^{(t+0.5\Delta t)}
\end{align*}
\]  

(6.8)  

(6.9)

where the superscripts \( (t - 0.5\Delta t) \) and \( (t + 0.5\Delta t) \) refer to mid-time increment values. Since the central difference operator is not self-starting because of the mid-increment of velocity, the initial values at time \( t = 0 \) for velocity and acceleration need to be defined. In this case, both parameters are set to zero as the disc remains stationary at time \( t = 0 \).

As opposed to the implicit dynamic integration (ABAQUS/Standard), explicit dynamic integration (ABAQUS/Explicit) does not need a convergent solution before
attempting the next time step. Each time step is so small that its stability limits $\Delta t$ is bounded in terms of the highest eigenvalue ($\omega_{\text{max}}$) in the system:

$$\Delta t \leq \frac{2}{\omega_{\text{max}}}$$ (6.10)

This is the reason for efficiency in ABAQUS/Explicit. Due to this “explicit” solution procedure for dynamic simulations, contact algorithms in explicit and implicit (read Standard in ABAQUS) versions are different. The main differences between the two solvers are given as follows:

- The Standard solver uses a pure master-slave scheme to enforce contact constraints while explicit solver, by default, uses a balanced master-slave scheme. Therefore alteration in explicit code is needed by changing a weighted average of contact constraints from WEIGHT = 1.0 (balanced) to WEIGHT = 0.0 (pure).

- The Standard solver and explicit solver both provide small sliding contact scheme. However, the small sliding scheme in standard solver transfers the load to the master nodes according to the current position of the slave nodes while in explicit solver, the small sliding scheme always transfers the load through the anchor point.

- The Standard solver, by default, uses a penalty contact algorithm to enforce frictional contact constraints while explicit solver, by default, employed a kinematic contact algorithm. The kinematic scheme applies sticking constraints in similar way to the Lagrange multipliers in standard solver. However, the algorithm is quite different. In order to change this default in explicit solver to the penalty method, an alteration in explicit code is needed by adding a keyword, MECHANICAL CONSTRAINTS = PENALTY.

Those considerations need to be taken into account so that when performing the complex eigenvalue and transient analysis, both analysis methods use similar contact algorithm and scheme. This is important when comparing results from those two analyses.
6.4.2 Disc Brake Assembly Model

Due to limitations in explicit version, the full disc brake model that is previously developed needs to be reduced by a number of components. The first limitation is due to unavailability of a specific spring element, which is used to connect one component to another for contact interaction. Thus, to avoid this problem as well as to reduce computational time in dynamic analysis, only the disc and two brake pads are considered, as illustrated in figure 6.23. The second limitation is that only reduced-integration 8-nodes solid element is available in explicit version, while full-integration 8-nodes solid element is used in standard version for the complex eigenvalue analysis.

![Figure 6.23: A reduced disc brake model](image)

The disc is rigidly constrained at the boltholes as before. For the pads, the leading edge is rigidly constrained in the circumferential direction while the trailing edge is constrained both at the circumferential and at the radial directions. Instead of applying a brake-line pressure on top of the piston head and of calliper housing, the pressure is now applied directly onto the back plates of the piston and finger pads. For the transient analysis, the time history of the brake-line pressure and rotational speed are used for describing operating conditions of the disc brake model as shown in figure 6.24. At the first stage, a brake pressure is applied gradually until it reaches $t_1$ and then it becomes uniform. The disc starts to rotate at $t_1$ and gradually increases up to $t_2$. Then the rotational speed becomes constant. For the complex eigenvalue analysis, the simulation procedure is described in chapter 3.
6.4.3 Simulation Results

In predicting instability of a system of interest, Hu et al (1997) used explicit dynamic finite element analysis with penalty method. Chargin et al (1997) used implicit dynamic finite element analysis with Lagrange multiplier to study aircraft disc brake instability. Baillet et al (2005) employed explicit dynamic analysis with similar contact constraints to study disc brake squeal. In this study three different contact schemes are simulated. This is performed in order to find out which contact scheme provided in ABAQUS can predict similar results to complex eigenvalue analysis and dynamic transient analysis. For this investigation, a constant friction coefficient is used while the disc brake is imposed by a brake-line pressure of 0.81 MPa and a rotational speed of 3.22 rad/s.

6.4.3.1 Finite sliding with Lagrange multiplier

The first contact scheme is using finite sliding with Lagrange multiplier. Using the complex eigenvalue, three unstable frequencies are predicted at 1.8 kHz, 2.8 kHz and 8.5 kHz as shown in figure 6.25. For the dynamic transient analysis, displacement in the z-direction at a particular node of the brake pad shows that its response decays...
with time until it level off at $t = 0.05$ s. This seems to suggest that the disc brake is stable throughout the analysis. Figure 6.26 illustrates response of the displacement. This time history response then is converted to the frequency domain using fast Fourier transformation through Matlab code. Frequency spectrum of the disc brake is plotted in figure 6.27. Even though there appears to be a high peak of frequencies at several locations, these frequencies are actually those of the stable system. Thus, it is suggested that both analyses predict different vibration behaviours of the system as the complex eigenvalue analysis most likely to show unstable system at three frequencies while the transient analysis tends to show a stable system throughout the analysis.

![Figure 6.25: Prediction of unstable frequencies using finite sliding with Lagrange multiplier](image)

Figure 6.25: Prediction of unstable frequencies using finite sliding with Lagrange multiplier

![Figure 6.26: Time history of z-displacement at a particular node for finite sliding with Lagrange multiplier](image)

Figure 6.26: Time history of z-displacement at a particular node for finite sliding with Lagrange multiplier
6.4.3.2 Finite sliding with penalty method

The second contact scheme is considering finite sliding with penalty method. Under this scheme, the complex eigenvalue analysis shows a similar result with the previous scheme where three unstable frequencies are predicted as depicted in figure 6.28. This may suggest that there is no difference between these two contact schemes in the complex eigenvalue analysis. However, the displacement response over time using this scheme is slightly different from that of figure 6.26 obtained using the previous scheme. Time history of the displacement is shown in figure 6.29. The figure shows that at time \( t = 0.05 \) s the displacement seems to grow up but not in significant amount. Thus, it is thought that this response did not represent instability in the system. The frequency spectrum of stable disc brake is shown in figure 6.30. Once again, there is some difference between the two analyses using this contact scheme.
Figure 6.28: Prediction of unstable frequencies using finite sliding with penalty method

Figure 6.29: Time history of z-displacement at a particular node for finite sliding with penalty method
6.4.3.3 Small sliding with Lagrange multiplier

The last contact scheme considered in this investigation is a small sliding with Lagrange multiplier. From figure 6.31, it is shown that there are two unstable frequencies predicted by the complex eigenvalue analysis, i.e. at frequencies of 1.8 kHz and 8.5 kHz which are characterised by 3-nodal diameters and in-plane unstable modes as shown in figure 6.32. In the transient analysis, the disc brake is likely to become unstable since the displacement response as illustrated in figure 6.33a is growing with significant amount and hence describes a divergent response after \( t = 0.01 \) s. Figure 6.33b shows acceleration response in time history of the selected node and this indicates that this instability occurred in a periodic limit-cycle and but not in the entire time history. This finding agrees with previous observations by Nack (2000). After converting displacement response in time domain into frequency domain, it is found that unstable frequencies are predicted at 1.8 kHz and 2.0 kHz with high vibration magnitude as depicted in figure 6.34. This agreed well with one of the two unstable frequencies predicted in the complex eigenvalue analysis. However, the other unstable frequency predicted by the complex eigenvalue analysis was not found in the transient analysis. This is similar to the case of Massi and Baillet (2005) where their transient analysis was able to predict only one of the two unstable frequencies found in the complex eigenvalue. From those three contact schemes, it is suggested that small sliding with Lagrange multiplier can predict more reasonable results than the other two contact schemes where displacement response

Figure 6.30: Predicted unstable frequencies for finite sliding with penalty method
shows instability of the disc brake while the other contact schemes show convergence in the displacement which means stable system whereas in the complex eigenvalue analysis always shows an unstable system with two or three unstable frequencies.

![Figure 6.31: Prediction of unstable frequencies using small sliding with Lagrange multiplier](image)

![Figure 6.32: Unstable mode shapes of the disc/pads at frequencies of 1.8 kHz (left) and 8.5 kHz (right)](image)
Figure 6.33: Time history of z-displacement (top) and z-acceleration (bottom) at a particular node for small sliding with Lagrange multiplier.

Figure 6.34: Predicted unstable frequencies for small sliding with Lagrange multiplier.
From predicted results it is suggested that complex eigenvalue analysis can give almost similar unstable frequencies for different contact schemes. This is due to the fact that predicted contact pressure distributions are nearly the same as shown in figures 5.0 and 5.1. Given almost the same contact pressures and put them into Eq. 3.12, it is not surprising that predicted unstable frequencies are almost identical. However, this is not the case for transient analysis where the disc is physically rotated (without considering moving load effect). Due to dissimilarities between finite and small sliding, predicted contact pressure or contact force is different. This can be seen from figure 6.35 that predicted total normal contact force for finite sliding with Lagrange or penalty gives nearly identical response over time history. However, normal contact force for small sliding with Lagrange multiplier shows differently where the response grows up with increasing time history. This may explain why both finite sliding with Lagrange or penalty method produce different stability behaviour compare to small sliding with Lagrange multiplier which shows unstable behaviour in the disc brake assembly.

![Figure 6.35: Comparison of total normal force predicted at the finger pad for different contact schemes](image)

**Figure 6.35:** Comparison of total normal force predicted at the finger pad for different contact schemes

### 6.5 Summary

This chapter has focused on the stability of the disc brake mainly using the complex eigenvalue analysis. Prior to the analysis, friction coefficient is calculated based on the developed mathematical model. Stability analysis is performed under different
friction characteristics including the effect of friction damping. It is shown that the best prediction result comes from the real contact interface model with the inclusion of friction damping and negative $\mu - \nu$ slope. The results also demonstrate that more unstable frequencies are predicted in the perfect contact interface model than the real contact interface model. Validation of the prediction results is not only based on its unstable frequencies but also in terms of its unstable mode shapes. Given part of the braking operating conditions, the predicted results show that there is good agreement in mode shapes between predicted and experimental results.

Stability analysis of the disc brake using different levels of disc brake modelling shows that the current model can predict unstable frequencies much better than the other models. Using a model with the disc and a pair of brake pads seems to lose one important unstable frequency. Neglecting only the carrier, the model is most likely to cover those squeals frequencies. However, more unstable frequencies are predicted.

Effect of wear on squeal occurrence is simulated. From the results, it shows that fugitive nature of squeal behaviour might due to wear. The wear simulations show that the unstable frequency is emerging and disappeared with a particular surface topography. However, classification on surface topography as proposed by some researchers is not considered in this work.

There is a question on how well the complex eigenvalue analysis and dynamic transient analysis correlate to each other in a large degrees of freedom model. In this work, using a similar disc brake model, boundary condition, operating condition and apparently similar contact scheme under a single software package, the results seem to show that with small sliding and Lagrange multiplier contact scheme the transient analysis can predict very well one of the unstable frequencies. However, another one as unstable frequency predicted in the complex eigenvalue analysis is missing from the transient analysis.