Laboratory Report Writing

E. M. Smith
A. H. Abdullah

Summary

Good report writing brings a direct reward in terms of promotion during the early stages of an engineer's career in industry. The technique of producing good reports is discussed in some detail.

In experimental work the method of keeping laboratory records required by the author is specified. A few of the experiments undertaken are required to be fully written up and presented in the form of high quality formal reports.
# Contents

**Abstract**

**Contents** 

**List of Figures**

**List of Tables**

**Nomenclature**

1 INTRODUCTION

2 FEATURES OF A REPORT

2.1 Types of Report

2.2 Designing a Report

2.3 Report Format

2.4 The Title

2.5 The Summary

2.6 The Contents List

2.7 Nomenclature

2.8 The Body of the Report

2.8.1 Introduction

2.8.2 Theory

2.8.3 Apparatus

2.8.4 Test Procedure

2.8.5 Observations and Calculation of Results

2.8.6 The Discussion

2.8.7 The Conclusion

2.8.8 The Acknowledgement

2.8.9 The References or Bibliography
Lists of Figures & Tables

List of Figures

1. Measurement of cooling water flow and heat from compressor. .......... 6
2. Variation of tensile stress with time for tie-rod number 1. ............... 7
3. Semi-log graph paper with 6 cycles on the $x$-axis. ....................... 9
4. Semi-log graph paper with simplified background. ....................... 10
5. (a) Theoretical curve. (b) Curve fitted to experimental values. .......... 11
6. Saturation temperature of steam $T$ at various pressures $p$. .......... 20
7. Graphical interpretation of error formula. ................................. 21
8. Error in the temperature versus pressure plot. ............................ 22

List of Tables

1. Flow measurement by Venturi meter. ................................. 6
Nomenclature

Roman Alphabets

\( A_e \) Volute cross-sectional area at exit (m\(^2\))
\( A_o \) Volute cross-sectional area at tongue clearance (m\(^2\))

Greek Alphabets

\( \beta \) Blade angle
\( \gamma \) Specific weight of fluid (N/m\(^3\))
\( \rho \) Density of fluid (kg/m\(^3\))

Subscripts

\( m \) meridional component
\( t \) tangential component
1 impeller inlet
1 INTRODUCTION

In the early stages of his/her graduate career in industry, the promotion of an engineer depends to a large extent on whether he can bring his ability to the attention of his supervisor. A manager is too busy and has too many subordinates to be able to assess by personal contact each engineer’s work and ability, and practically the only way he has of gauging merit is through the written reports which arrive on his desk. The engineer who produces an easily read, accurate and lucid report of the work he is doing will immediately gain recognition.

It is not easy to write a good report—it combines technique and hard work—the technique can be learned, hard work is the lot of the engineer.

At University level a student’s report is read, because this is one of the duties of his/her instructor. In industry or in research there is no obligation on the part of a supervisor to read any report prepared by his subordinates. Thus, if advantage is not taken of the opportunity to learn to improve presentation at University level, future promotion in a chosen career may well be as effectively blocked as when dead-end jobs are taken.

At the University, laboratory experiments are to be regarded as providing opportunities to practice certain aspects of the writing of successful reports as well as demonstrations of scientific facts and experience in experimental technique.

2 FEATURES OF A REPORT

2.1 Types of Report

The writing of a paper for a learned journal, or of an article for a technical magazine will not be discussed here. Individual publishers have their preferred techniques of presentation and an intelligent author will study the structure of papers already published in the journal of his choice, before submitting his manuscript for review.

Not every engineer can aspire to achieve publication in a technical journal, but every engineer will be called upon at some time in his career to write an internal report for consumption within his firm. It is the internal report with which he will mostly be concerned here. The structure of such a report is given in Appendix A.

Reports, like papers, are written in the third person.
2.2 Designing a Report

When designing a report the reason for writing it has to be kept in mind by the author, e.g. it may be to inform or to recommend a course of action: a report is generally aimed at a particular person. When deciding the depth of his report the author must consider the nature of the interest and the extent of the knowledge of the intended reader.

The probable actions of the prospective reader should be taken into account. He will look at the title, note the name of the author and then flip through the report to see “what he is in for”. He looks for and interprets (perhaps not very consciously), any clues which help him to decide what the document is about, trying to estimate how carefully and how much of it he will need to read, what degree of urgency it demands and what risk he runs by not reading it. (Should the student be tempted to consider that the risk is small in not reading this report, he should look at conclusion 3, 4 and 5).

Clues as to quality are generally looked for in the summary, the conclusions and in the illustrations, and possibly in the introduction. A well laid-out report with structural headings permits rapid location and evaluation of these clues, and when the clues are properly constructed the reader is encouraged to proceed to greater depth.

2.3 Report Format

A favourable impression is created when the presentation of the report is neat and tidy. This is most easily achieved when a set of simple conditions is fulfilled.

The text of reports is to be written on a good quality plain white A4 (297 mm × 210 mm) paper on one side only. Graph paper or other special purpose paper is to be of the same size.

The finished report is to be stapled down the left hand side, so that it is essential to leave a margin of 25 mm on each page, including any pages of graphs, tables or illustrations.

The first page will carry out the title followed by the name of the author about 50 mm from the top, and the summary about midway down the page. The date will appear at the bottom of the page.

The second page will carry only the list of contents complete with page numbers. The Dewey decimal system of numbering is to be adopted.

The third page will start with the introduction and subsequent sections will follow naturally without necessarily taking a new page at each heading until the acknowledgements have been completed.

A new page will be taken for the references.
A new page will be taken for nomenclature.

Figures, diagrams, graphs and tables will appear on separate pages, but more than one item of the same kind (e.g. three diagrams, two tables) are permissible on a single page if space permits.

A new page will be taken at the start of the appendices.

2.4 The Title

The title has to be fairly short so that it is easily remembered. It is difficult, but not impossible, to write a short sentence not exceeding 10 words which describes a piece of work accurately.

The dangers are:

(a) to make the title too long and descriptive so that it cannot be remembered.

(b) to make it short, but couched in such general terms that it does not describe accurately the subject of the report.

2.5 The Summary

The summary is a condensation of the whole report, not a convenient repository for afterthoughts, excuses, explanations, etc. Its purpose is to amplify the title of the report and to indicate, to those with some knowledge of the context, the scope and main conclusions of the report.

The summary should be written after the report is complete, and it should not exceed 100 words.

2.6 The Contents List

The contents list is prepared at least twice, once at the start so that the author obtains an overall picture of what he is going to say and again immediately before the summary is written, i.e. after the report is complete. It may be rewritten several times in between as the author reorganises his thoughts on presentation. The Dewey decimal system is one of the most convenient methods of numbering sections and subsections, e.g.

1. SECTION A
2. SECTION B
3. SECTION C
3.1. Sub-section C1
3.2. Sub-section C2
4. SECTION D...........etc.

Note that the sub-heading 3.1. Sub-section C1 follows immediately on the heading 3. SECTION C. No text is permitted between these two titles; if text is included it would also require a sub-heading.

2.7 Nomenclature

When theoretical work is included in a report, it is advisable to include a nomenclature explaining the symbols used. The nomenclature should be placed at the beginning of the report so that it is easily located. (It may sometimes be placed at the end).

A convenient way of listing symbols is to group them into Roman Alphabets, Greek Alphabets and Subscripts. Within an alphabetic group the symbols should be listed in alphabetic order, with lower case letters proceeding capitals, e.g.

Roman Alphabets

\[ A_e \quad \text{Volute cross-sectional area at exit (m}^2\text{)} \]
\[ A_o \quad \text{Volute cross-sectional area at tongue clearance (m}^2\text{)} \]
\[ \ldots \]

Greek Alphabets

\[ \beta \quad \text{Blade angle} \]
\[ \gamma \quad \text{Specific weight of fluid (N/m}^3\text{)} \]
\[ \rho \quad \text{Density of fluid (kg/m}^3\text{)} \]
\[ \ldots \]

Subscripts

\[ m \quad \text{meridional component} \]
\[ t \quad \text{tangential component} \]
\[ 1 \quad \text{impeller inlet} \]
\[ \ldots \]

2.8 The Body of the Report

No firm rule exists for selecting appropriate headings in the body of the report. Common sense will prevail, but such headings may include theoretical development and/or exper-
imental work, etc. Experimental work may best be handled in a sequence of sub-sections entitled apparatus, test procedure and observations and calculation of results.

No work is scientific unless sufficient information is given to permit another scientist to repeat the experiments and make the same observations. A scientific worker must therefore describe the steps in his research in logical sequence.

2.8.1 Introduction

The introduction should explain the problem, and state the purpose of the work in clear and concise terms. It may include brief consideration of previous work in the form of a short historical review so as to place the report correctly in its related field of work. The interests, attitudes and knowledge of the intended reader must be taken into account.

2.8.2 Theory

The main theoretical argument should be outlined, but detailed mathematical development is best placed in an appendix. Be sure that the appendices are numbered, and that the reader is referred in the text to the appropriate section.

2.8.3 Apparatus

Apparatus is best described in conjunction with a simple, clearly labelled (e.g. single-stage reciprocating compressor, two-way valve etc.), numbered (e.g. Figure 1) and captioned (e.g. Measurement of cooling water flow and heat from compressor) diagram.

Remember it is the principle of the apparatus which is important, and not facts such as the use of a 20 mm bore stop valve at a tank outlet.

The functions of special equipment should be described in the body of the report under Apparatus, with perhaps a special sub-heading if this is appropriate. If detailed description of such equipment is lengthy it may be placed in an appendix and appropriate reference to it made in the text. Descriptions of standard machines and instruments, other than a clear statement of their general nature, range etc. are not, in general, required.

2.8.4 Test Procedure

The past tense should be used, as work which has been completed is being described. Steps taken should be described in logical order. A clear statement of any particular precautions taken is essential, and details, e.g. of load increments etc. should be given.
2.8.5 Observations and Calculation of Results

Experimental observations include the recording of test data such as temperature, pressure etc., and of factual statements as to the functioning or mal-functioning of the apparatus.

Experimental data and the derived results should be neatly tabulated whenever possible. Units must be clearly indicated, and the layout should be such that it is possible to distinguish at a glance between observed (i.e. experimental) and derived (i.e. calculated) values—a specimen of each calculation should be given, and where this is lengthy it should be referred to in the body of the report and described in detail in the appendix. Every table should have a title and a table number (see Table 1) so that it may be referred to in the text.

**Table 1:** Flow measurement by Venturi meter.

<table>
<thead>
<tr>
<th>Run No</th>
<th>Head $h$ (mm H$_2$O)</th>
<th>Time for 50 l $t$ (s)</th>
<th>$\sqrt{h}$</th>
<th>Theoretical flowrate $Q_i$ (m$^3$/s)</th>
<th>Measured flowrate $Q_m$ (m$^3$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37.2</td>
<td>32.0</td>
<td>6.10</td>
<td>0.250</td>
<td>0.252</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>…</td>
<td></td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
</tbody>
</table>

Comment on and interpretation of the observations should be reserved for a section headed **Discussion**.
2.8.6 The Discussion

Attention should be drawn to the salient experimental results. The effects of test variables and the significance of such affects should be discussed. Where applicable, comparisons should be made with the theory, see Figure 2, and explanations offered for deviations from the theory. Mention should be made of any side effects, faults or uncontrolled variables noticed during the tests which might affect the results. If such results are to be of use, absolute honesty is essential.

Further topics might include: sources of experimental error with an estimate of their quantitative effect on the results (detailed mathematical analysis would be placed in an appendix); criticisms of experimental methods and suggestions for improvement; possible further work which could throw more light on the topic of investigation.

![Figure 2: Variation of tensile stress with time for tie-rod number 1.](image)

2.8.7 The Conclusion

The presence of a conclusion helps to assure the reader that a report has been planned and finished. It should consist of a numbered list of firm statements or recommendations with the minimum of qualifications providing definite answers to problems set out in the Introduction.

2.8.8 The Acknowledgement

Where other persons have assisted substantially in any part of the subject of the report, they should be acknowledged.
It is also polite to thank the head of the firm or department who permitted the use of his organisation’s time, equipment and money. In laboratory work at the University the assistance of your colleagues in gathering the data should be acknowledged, e.g.

*I would like to record my special thanks to technicians in the Fluid Mechanics Laboratory for making available facilities for us to carry out the experiment, …and to all members of my group, thank you so much for your valuable suggestions and many sessions of late night discussion.*

2.8.9 The References or Bibliography

Information which has already been published and which is readily available in libraries should not be included in a report. Instead the exact reference thereto should be given, including:

(a) Authors name and initials.
(b) Title of publication if it is a book.
(c) Title of article and name of journal if it is a paper.
(d) Publishers name.
(e) Edition, date and place of publication if it is a book.
(f) Volume number, part number, issue number and date of publication if it is a journal.

It is impossible to give too much information.

For convenience references are generally listed at the end of a paper, in the order that they are referred to in the paper, and not in alphabetic order.

Points concerning the correct use of references may be found in references (1) and (2).

2.8.10 Figures, Tables and Graphs

Each figure, graph and table must carry a number (e.g. Figure 1, Table 1) and a caption. Conventional symbols are to be used when representing standard pieces of equipment (e.g. the reciprocating compressor in Figure 1). When no appropriate symbol is defined, then the diagram should represent the apparatus in a simplified and easily understood style.

Figures must be *keyed* to the text by labelling, so that description of experimental procedure is clear and unambiguous and the correct sequence of steps is evident.
Pictorial sketches of engines, turbines, testing machines etc. enhance the appearance of a report. However such illustrations are not essential if the machine make, type and serial number can be specified and an exact reference to a handbook can be given. If non-standard equipment is fitted to an engine or machine, it may sometimes be worthwhile to include an outline sketch of the engine or machine showing the special equipment in full detail placed in the correct position.

When tabulating a result during calculation, remember that the answer cannot be more accurate than the original data. It may however be less accurate. Values should generally be tabulated as decimals, e.g. a time of 1 hour 12 minutes should be written as 1.20 hours, or 72.0 minutes.

The decimal system is Universal in science and the choice of scales should recognise this fact, i.e.

$$0, 5, 10, 15 \ldots$$
$$0, 2, 4, 6, 8, 10 \ldots$$
$$0, 0.01, 0.02, 0.03, etc.$$ are acceptable, but

$$0, 3, 6, 9 \ldots$$
is not.

When plotting a graph it is unusual for more than three significant figures to be required. In certain special circumstances greater accuracy may be required and 6-figure tables should then be used. Each scale of graph must be labelled, e.g. **Pressure**, and be provided with dimensions e.g. N/m$^2$.

![Figure 3: Semi-log graph paper with 6 cycles on the x-axis.](image-url)
Special purpose graph papers (e.g. polar co-ordinate, log/log, linear/log as in Figure 3) are sometimes used for plotting of experimental results. They are, however, generally not suitable for inclusion in a formal report as there is too much background. The log axis of Figure 3, for instance, runs in exponential cycles. Each cycle runs linearly in 10’s but the increase from one cycle to another is an increase by a factor of 10. So within a cycle you would have a series of: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 (this could also be 1–10, or 0.1–1.0, etc.). The next cycle actually begins with 100 and progresses as 200, 300, 400, 500, 600, 700, 800, 900, 1000. The cycle after that would be 1000, 2000, ... 10000. The semi-log graph paper in Figure 3 has 6 cycles on the x-axis. Note that when logarithmic scales are used the representation of zero is not possible.

To accomplish its purpose in a formal report, the background grid on which the curves are drawn has to be simplified as shown in Figure 4. For University work however, commercially ruled paper will be acceptable.

![Semi-log graph paper](image)

**Figure 4:** Semi-log graph paper with simplified background.

Theoretical curves should be represented as smooth curves, see Figure 5(a). Do not indicate the points used to construct the theoretical curve. Inspect the theoretical formula to see if the curve will pass through the origin, and if it does try to include the origin where this is convenient.

Experimental values should be represented as individual points, using small circles, crosses, squares, etc., to identify different curves. Such experimental values may be correlated by drawing the best smooth curve through the points and this curve need NOT pass through all or any of the points, as shown in Figure 5(b).

Experimental values remote from the smooth curve may have a thin line, normal to the smooth curve, connecting them to the smooth curve so as to indicate their belonging.
Further details concerning the presentation of graphs may be found in references (1) and (2).

2.8.11 The Appendix

Any relevant detailed material, e.g., experimental results, statistical data, mathematical derivations, etc., which if included in the body of the report would hinder development of the main thesis, should be put in an appendix. It can there be read by those who wish to read it, accepted or ignored by those who do not.

An appendix should be as well written as the main body of the text, e.g., it should not degenerate into note form.

3 Laboratory Records

On average a student will attend laboratory classes once a week for twelve weeks. A working record of each experiment undertaken should be kept in an A4 size soft cover laboratory logbook.

The object of the record is to keep a brief account of the investigation and the results obtained, in sufficient detail to permit amplification to a full report at some later date if required. (This is normal practice in any piece of scientific or engineering work). This rough record is for the student’s benefit and ought to be completed during the laboratory period and should contain:
(a) the title and date of the experiment,
(b) the names of the investigators,
(c) a concise statement of test procedures,
(d) a sketch of the apparatus, showing the location of all sensing points for the instrumentation,
(e) all observations and calculated results in tabular form,
(f) comments on difficulties encountered with suggested means of overcoming them.

The high standard of a formal report is not expected with laboratory records, but tidiness is essential. Logbooks may be inspected at any time should the formal reports submitted prove unsatisfactory.

4 SUBMISSION OF REPORTS

Two formal reports complete with estimations of error are required during each term of the laboratory course; Appendix B introduces the estimation of experimental errors. The first report will be written on the first experiment undertaken, and is to be submitted not later than 14 days (including weekend) after completion of the experiment. The maximum permitted length of each report is 15 pages. It must be enclosed in official blue covers and black plastic spine for student reports, obtainable from the Librarian of the Faculty of Mechanical Engineering.

The experiment to be the subject of the further report will be indicated after the first report has been assessed and discussed with each student individually. In each report an estimation of error not exceeding 2 pages in length is required.

The standard required in the formal reports is high. A well written report has usually been redrafted about five times before reaching its final form. In industry it is never practicable to omit the first four drafts if a report of real quality is to emerge. The general style outlined in this document is to be adopted. Two indispensible aids to good writing are a dictionary (e.g., Reference 5) and Roget’s Thesaurus (Reference 6). The latter book helps an author to find the word appropriate to a given situation when he can only think of the required meaning.

Corrections, deletions, ink blots, fingermarks, tea-stains, etc., have no place in the reports required. Reports in untidy handwriting and late reports will not be assessed. If a sufficiently high standard of appearance and presentation is not attained, the student may be required to rewrite part or all of a report to achieve the desired standard.
Students are reminded that failure to submit satisfactory laboratory work may result in the defaulter’s name being referred to the Registrar, who may then not permit that student to sit any examination.

5 CONCLUSIONS

- The incentives to good report writing have been discussed.
- The features essential to a successful report have been examined in detail, attention being focussed on the effect of the report on the reader.
- The method of keeping laboratory records during experimental work has been indicated.
- The requirements for submission of formal laboratory reports of high standard have been specified.
- The penalty for failure to submit work of sufficient quality on time has been clearly stated.

Acknowledgements

The authors wish to thank those persons whose advice, criticism and suggestions have contributed to this report.
Bibliography

A STRUCTURE OF AN EXPERIMENTAL REPORT

A report will generally have a structure similar to that given below:

1. A Title (not exceeding 10 words)
   The Author’s Name
   A Summary (not exceeding 100 words)

2. List of Contents

3. List of Figures

4. Nomenclature for Symbols Used

5. Introduction

6. Theoretical Development

7. Experimental Work
   7.1 Apparatus
   7.2 Procedures
   7.3 Observations and Calculation of Results

8. Discussion of Results

9. Conclusions (listed numerically)

10. Acknowledgements

11. References or Bibliography

12. Appendices (if appropriate)
B ESTIMATION OF EXPERIMENTAL ERRORS

B.1 Introduction

In research it is never good practice to make an experiment until at least some theoretical studies have been completed. Even if the theory is only approximate, it will help in designing the tests to be made.

Once the rig is running, experimental results can be compared with the theoretical predictions, and improvements in both will follow.

B.2 Types of Experiment

We can distinguish two types of experiment

a. that in which a number of variables (say \( x, y, z \)) are measured experimentally, and then combined in some mathematical formula to produce a result (say \( V \)) by calculation,

b. that in which we wish to establish the relationship between two experimentally measured variables, for example the temperature \( T \) and the pressure \( p \) of a saturated liquid.

B.3 Sources of Error in Individual Measurements

In any measurement experimental errors arise due to

i. the inaccuracy of the measuring device,

ii. the inability of the experimenter to observe with absolute precision,

iii. the occurrence of unavoidable transient effects during the experiment.

Errors due to (i) can of course be reduced but not eliminated, if the measuring device can be calibrated by a more accurate instrument or, ultimately, against some primary standard. Note however that the inaccuracy in the device can never be less than that of the calibrating instrument, and that eventually the accuracy of the primary standard itself must be established, thus involving analysis of type (b) experiments.

In undergraduate work it is not practicable to calibrate every device used in an experiment, and the problem of estimating the magnitudes of errors then arises. As an example, consider the use of a Bourdon Tube Pressure gauge with a scale reading from 0–50 N/m\(^2\)
abs. Assuming that the instrument is not faulty, the safest procedure is to consult the appropriate British Standard (available in the Main Library) and use the specified accuracy; another alternative but less precise technique is to look at the spacing of the scale markings and estimate how accurate the manufacturers considered his instrument to be by this somewhat circumstantial evidence.

Errors due to (ii) may be caused by parallax error due to the observer not standing directly in front of an instrument in which the pointer of the instrument is significantly distant from the scale. Type (ii) errors may also arise if reading the instrument requires interpolation of large scale divisions. The method of allowing for such errors should be obvious.

Errors due to (iii) usually involve thinking about the physical properties (e.g. mechanical, electrical, thermal, magnetic etc.) of the sensing transducer of its associated measuring instrument. Errors are reduced (but not eliminated) by minimising mertios, e.g. by using very fine thermocouple wires instead of mercury in glass thermometers for temperature measurement. In very accurate work, sophisticated theoretical analyses may be used to estimate the true temperature from measured values, but discussion of such advanced techniques is not appropriate at this stage, for each case requires individual mathematical treatment.

Suppose, in the work with which we are concerned, we are measuring a variable $x$. Then errors may exist

\[
\pm \delta x_1 : \text{arise due to (i)} \\
\pm \delta x_2 : \text{arise due to (ii)} \\
\pm \delta x_3 : \text{arise due to (iii)}
\]

Combining these we write

\[
\delta x = |\delta x_1| + |\delta x_2| + |\delta x_3| \tag{1}
\]

so that the true value of $x$ will lie in the range $x \pm \delta x$.

### B.4 Error Analysis in Type (a) Experiments

The procedure here is straightforward. We examine mathematically how errors $\pm \delta x$, $\pm \delta y$, $\pm \delta z$ in variables $x$, $y$, $z$ say, propagate through a given mathematical formula. To illustrate the procedure, consider two examples.

**Example 1**
ESTIMATION OF EXPERIMENTAL ERRORS

Given the formula

\[ V = x + y - z \]  \hspace{1cm} (2a)

and assuming errors of \( \pm \delta x \) in \( x \), \( \pm \delta y \) in \( y \), \( \pm \delta z \) in \( z \) what is the error in \( V \)?

From Eq. 2a

\[ V = x + y - z \]

thus

\[ V \pm \delta V = (x + \pm \delta x) + (y + \pm \delta y) - (z + \pm \delta z) \]

\[ = (x + y - z) \pm (\delta x + \delta y - \delta z) \]

so that

\[ \delta V = (\delta x + \delta y - \delta z) \]  \hspace{1cm} (2b)

and the fractional error would thus be in the range

\[ \frac{\delta V}{V} = \frac{(\delta x + \delta y + \delta z)}{x + y - z} \]  \hspace{1cm} (2c)

**Example 2**

Given the relationship

\[ V = \frac{x^ay^b}{z^c} \]  \hspace{1cm} (3a)

and assuming errors of \( \pm \delta x \) in \( x \), \( \pm \delta y \) in \( y \), \( \pm \delta z \) in \( z \) what is the error in \( V \)?

From Eq. 3a

\[ V = \frac{x^ay^b}{z^c} \]

thus

\[ V \pm \delta V = \frac{(x \pm \delta x)^a (y \pm \delta y)^b}{(z \pm \delta z)^c} \]

\[ = (x \pm \delta x)^a (y \pm \delta y)^b (z \pm \delta z)^{-c} \]
Now expand the brackets by the Binomial theorem

\[(x \pm \delta x)^a (y \pm \delta y)^b (z \pm \delta z)^{-c}\]

\[= \left( x^a \pm \frac{a}{1!} x^{a-1} \delta x \ldots \right) \left( y^b \pm \frac{b}{1!} y^{b-1} \delta y \ldots \right) \left( z^c \pm \frac{-c}{1!} z^{c-1} \delta z \ldots \right)\]

\[= \frac{x^a y^b}{z^c} \left( 1 \pm a \frac{\delta x}{x} \right) \left( 1 \pm b \frac{\delta y}{y} \right) \left( 1 \pm (-c) \frac{\delta z}{z} \right)\]

so that

\[\delta V = \frac{x^a y^b}{z^c} \left( a \frac{\delta x}{x} + b \frac{\delta y}{y} - c \frac{\delta z}{z} \right)\]  

(3b)

and the fractional error would thus be in the range

\[\frac{\delta V}{V} = a \frac{\delta x}{x} + b \frac{\delta y}{y} - c \frac{\delta z}{z}\]

Again, \(\delta z\) may be \(-ve\) when \(\delta x\) and \(\delta y\) are \(+ve\), so taking the worst case, the fractional error is in the range

\[\frac{\delta V}{V} = a \frac{\delta x}{x} + b \frac{\delta y}{y} + c \frac{\delta z}{z}\]  

(3c)

An alternative treatment would be to proceed by partial differentiation where, from Eq. 3a,

\[V = x^a y^b z^{-c}\]  

(4a)

and differentiating, w.r.t. \(x\), yields

\[\frac{\partial V_x}{\partial x} = a x^{a-1} y^b z^{-c} = \frac{a}{x} V\]

Separating variable,

\[\frac{\delta V_x}{V} = \frac{a}{x} \frac{\delta x}{x}\]

Similarly

\[\frac{\delta V_y}{V} = b \frac{\delta y}{y}\]  

and

\[\frac{\delta V_z}{V} = -c \frac{\delta z}{z}\]
Total error
\[ \frac{\partial V}{V} = \frac{\delta V_x}{V} + \frac{\delta V_y}{V} + \frac{\delta V_z}{V} \]  
\[ = a \frac{\delta x}{x} + b \frac{\delta y}{y} - c \frac{\delta z}{z} \]  
(4b)

Taking the worst case as before
\[ \frac{\partial V}{V} = a \frac{\delta x}{x} + b \frac{\delta y}{y} + c \frac{\delta z}{z} \]  
(4c)

Figure 6: Saturation temperature of steam $T$ at various pressures $p$.

B.5 Error Analysis in Type (b) Experiments

Suppose we wish to measure experimentally the saturation temperature $T$ of steam at various pressures $p$. We might use an apparatus such as that indicated in sketch, and plot a graph of $T$ versus $p$.

From the graph in Figure 6 we can say, mathematically, that $T$ is a function of the single variable $p$. However there are experimental errors in the observations made of both $T$ and $p$, and before we can state the accuracy of the results (i.e. the width of the likely scatter-band about the mean line through the observations) we must estimate the magnitude of type (i), (ii) and (iii) errors in both $T$ and $p$ from a knowledge of the care taken in constructing the apparatus and in carrying out the tests. This we shall now do.

Suppose the true (and as yet unknown) relationship between $T$ and $p$ is
\[ T = f(p) \]  
(5)
and we have estimated experimental errors of $\pm \delta T$ in $T$ and $\pm \delta p$ in $p$ on the basis of the mean of a number of repeated observations.

If at a particular point the recorded pressure $p$ is the pressure for which we wish to find the true saturation temperature $T_1$, then we must take into account the fact that the actual pressure in the apparatus lies in the range $p \pm \delta p$, so that there is an additional error $\delta \theta_1$ in $T$ due to the fact that $T$ is a function of $p$.

The total possible error in temperature is then $(\delta \theta_1 + \delta T_1)$.

From Eq. (5) we have $T = f(P)$.

The functional error in $T_1$ is thus

$$\delta \theta_1 = f(p_1 \pm \delta p_1) - f(p_1)$$

Expanding $f(p_1 \pm \delta p_1)$ by Taylor’s theorem

$$\delta \theta_1 = f(p_1) + \left( \frac{\delta p_1}{1!} \frac{\partial f(p_1)}{\partial p_1} + \frac{(\delta p_1)^2}{2!} \frac{\partial^2 f(p_1)}{\partial p_1^2} + \ldots \right) - f(p_1)$$

Neglecting second and higher order terms

$$\delta \theta_1 = \pm \delta p_1 \frac{\partial f(p_1)}{\partial p_1} = \pm \delta p_1 \left( \frac{\partial T}{\partial p} \right)_1$$

The total error in $T$ is thus

$$\pm (\delta \theta_1 + \delta T_1) = \pm \left[ \delta p_1 \left( \frac{\partial T}{\partial p} \right)_1 + \delta T_1 \right]$$

Figure 7: Graphical interpretation of error formula.

To put numerical values in this expression, we see that $\delta p_1$ and $\delta T_1$ are available from experimental work but that $(\partial T / \partial p)_1$ is the slope of the theoretical function $T = f(p)$ at the point 1. This we do not have, but a very good approximation to it can be obtained.
from the slope of the graph of the observed experimental results, Figure 7. The total error in $T_1$ can then be evaluated and plotted above and below the observed curve as indicated in Figure 8.

Figure 8: Error in the temperature versus pressure plot.